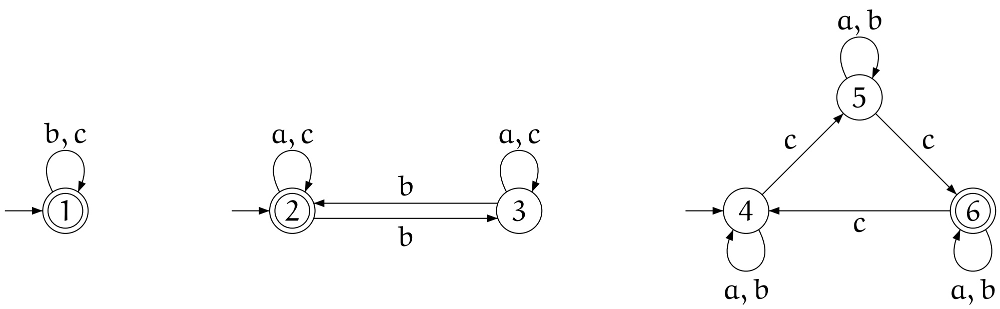
# Tutorial 3

**Exercise 1:** Construct NFA accepting the following languages:

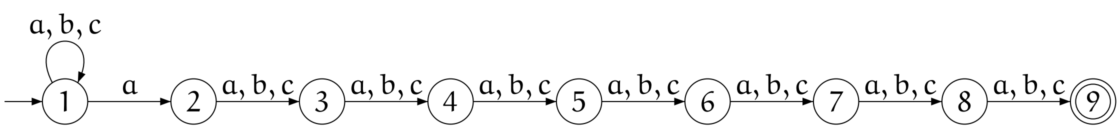
a) $L_1 = \{w \in \{a, b, c\}^* \mid |w|_a = 0 \vee |w|_b \bmod 2 = 0 \vee |w|_c \bmod 3 = 2\}$

*Solution:* The automaton could be easily constructed by combining three separate automata. Alternatively, we could add one new initial state with ε-transitions to the original three initial states (that need not be initial now).



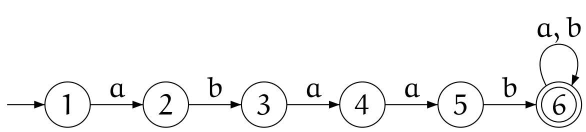

b) $L_2 = \{w \in \{a, b, c\}^* \mid |w| \geq 8$ and the eighth symbol from the end of word $w$ is $a\}$

*Solution:*



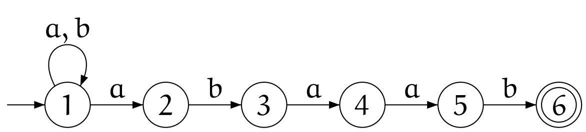

c) $L_3 = \{abaabw \mid w \in \{a, b\}^*\}$

*Solution:*



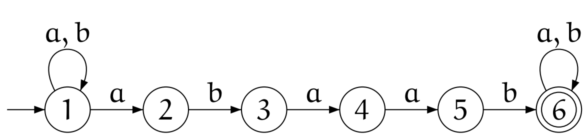

d) $L_4 = \{wabaab \mid w \in \{a, b\}^*\}$

*Solution:*

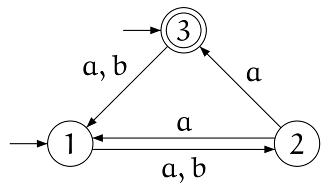



e) $L_5 = \{w_1abaabw_2 \mid w_1, w_2 \in \{a, b\}^*\}$

*Solution:*



**Exercise 2:** Construct a DFA equivalent to the given NFA:

a) 


*Solution:*

Original automaton:

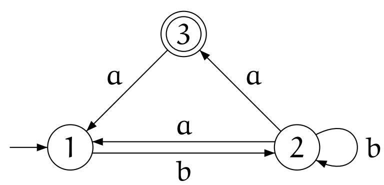
| | a | b |
|---|---|---|
| $\rightarrow$ 1 | 2 | 2 |
| 2 | 1,3 | – |
| $\leftrightarrow$ 3 | 1 | 1 |

Resulting automaton:

| | a | b |
|---|---|---|
| $\leftrightarrow \{1,3\}$ | $\{1,2\}$ | $\{1,2\}$ |
| $\{1,2\}$ | $\{1,2,3\}$ | $\{2\}$ |
| $\leftarrow \{1,2,3\}$ | $\{1,2,3\}$ | $\{1,2\}$ |
| $\{2\}$ | $\{1,3\}$ | $\emptyset$ |
| $\emptyset$ | $\emptyset$ | $\emptyset$ |

After renaming states:

| | a | b |
|---|---|---|
| $\leftrightarrow$ 1 | 2 | 2 |
| 2 | 3 | 4 |
| $\leftarrow$ 3 | 3 | 2 |
| 4 | 1 | 5 |
| 5 | 5 | 5 |

b) 


*Solution:*

Original automaton:

| | a | b |
|---|---|---|
| $\rightarrow$ 1 | – | 2 |
| 2 | 1,3 | 2 |
| $\leftrightarrow$ 3 | 1 | – |

Resulting automaton:

| | a | b |
|---|---|---|
| $\rightarrow \{1\}$ | $\emptyset$ | $\{2\}$ |
| $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $\{2\}$ | $\{1,3\}$ | $\{2\}$ |
| $\leftarrow \{1,3\}$ | $\{1\}$ | $\{2\}$ |

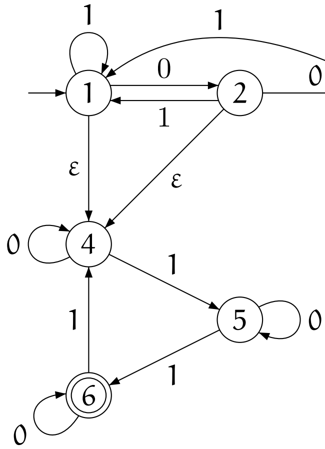
After renaming states:

| | a | b |
|---|---|---|
| $\rightarrow$ 1 | 2 | 3 |
| 2 | 2 | 2 |
| 3 | 4 | 3 |
| $\leftarrow$ 4 | 1 | 3 |

**Exercise 3:** Construct GNFA accepting languages $L_1$, $L_4$ and $L_5$:

a) $L_1 = L_2 \cdot L_3$, where
$L_2 = \{w \in \{0,1\}^* \mid \text{every occurrence of } 00 \text{ in } w \text{ is immediately followed by } 1\}$
$L_3 = \{w \in \{0,1\}^* \mid |w|_1 \bmod 3 = 2\}$

*Solution:*

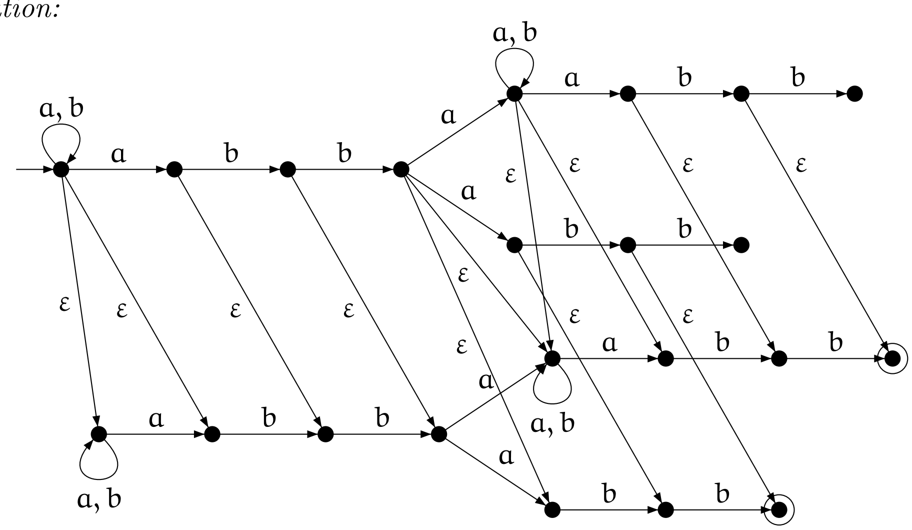



b) $L_4 = \{ w \in \{0,1\}^* \mid w$ contains at least three times subword $000 \}$

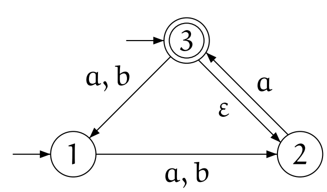
*Remark:* The occurrences of the subword can overlap, so the language L contains for example word 00000.

c) $L_5 = \{w \in \{a,b\}^* \mid w$ is obtained from some word $w' \in L_6$ by ommiting of one symbol$\}$, where $L_6$ is the language consisting of those words over alphabet $\{a,b\}$ that contain subword $abba$ and end with suffix $abb$.

*Solution:*




**Exercise 4:** Construct equivalent DFA for the given GNFA:

a) 


*Solution:*

Original automaton:

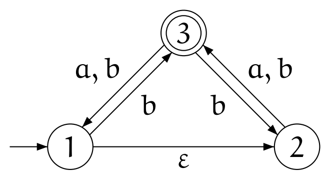
| | a | b | ε |
|---|---|---|---|
| → 1 | 2 | 2 | – |
| 2 | 3 | – | – |
| ↔ 3 | 1 | 1 | 2 |

Resulting automaton:

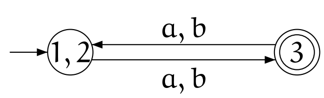
| | a | b |
|---|---|---|
| ↔ {1, 2, 3} | {1, 2, 3} | {1, 2} |
| {1, 2} | {2, 3} | {2} |
| ← {2, 3} | {1, 2, 3} | {1} |
| {2} | {2, 3} | ∅ |
| {1} | {2} | {2} |
| ∅ | ∅ | ∅ |

After renaming states:

| | a | b |
|---|---|---|
| ↔ 1 | 1 | 2 |
| 2 | 3 | 4 |
| ← 3 | 1 | 5 |
| 4 | 3 | 6 |
| 5 | 4 | 4 |
| 6 | 6 | 6 |

b) 


*Solution:*




Original automaton:

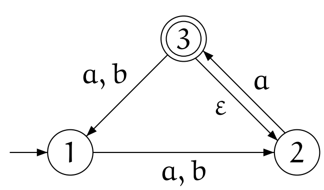
| | a | b | ε |
|---|---|---|---|
| → 1 | – | 3 | 2 |
| 2 | 3 | 3 | – |
| ← 3 | 1 | 1,2 | – |

Resulting automaton:

| | a | b |
|---|---|---|
| → {1, 2} | {3} | {3} |
| ← {3} | {1, 2} | {1, 2} |

After renaming states:

| | a | b |
|---|---|---|
| → 1 | 2 | 2 |
| ← 2 | 1 | 1 |

c) 


*Solution:*

Original automaton:

| | a | b | ε |
|---|---|---|---|
| → 1 | 2 | 2 | – |
| 2 | 3 | – | – |
| ← 3 | 1 | 1 | 2 |

Resulting automaton:

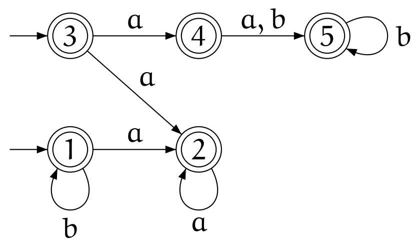
| | a | b |
|---|---|---|
| → {1} | {2} | {2} |
| {2} | {2, 3} | ∅ |
| ← {2, 3} | {1, 2, 3} | {1} |
| ∅ | ∅ | ∅ |
| ← {1, 2, 3} | {1, 2, 3} | {1, 2} |
| {1, 2} | {2, 3} | {2} |

After renaming states:

| | a | b |
|---|---|---|
| → 1 | 2 | 2 |
| 2 | 3 | 4 |
| ← 3 | 5 | 1 |
| 4 | 4 | 4 |
| ← 5 | 5 | 6 |
| 6 | 3 | 2 |

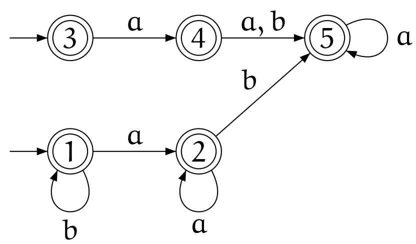
**Exercise 5:** For each of the following automata find at least one word over alphabet $\{a, b\}$, which is not accepted by the given automaton.

a)




*Solution:* For example $bab$.

b)

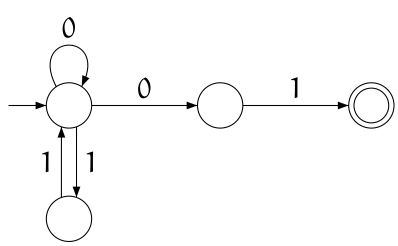



*Solution:* For example $abb$.

**Exercise 6:** For each of the following regular expressions, construct an equivalent finite automaton (it can be a GNFA):

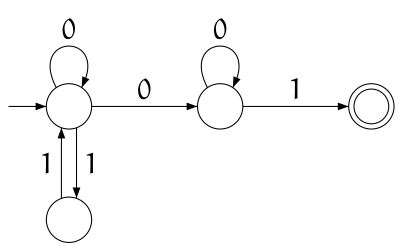
a) $(0 + 11)^*01$

*Solution:*




b) $(0 + 11)^*00^*1$

*Solution:*




c) $(a + bab)^* + a^*(ba + \varepsilon)$

**Exercise 7:** Describe an algorithm that for a given NFA $\mathcal{A} = (Q, \Sigma, \delta, I, F)$ decides if:

a) $\mathcal{L}(\mathcal{A}) = \emptyset$

*Solution:* $\mathcal{L}(\mathcal{A}) \neq \emptyset$ iff an accepting state can be reached from some initial state. (Reachable states can be easily computed, for example by breadth-first search.)

b) $\mathcal{L}(\mathcal{A}) = \Sigma^*$

*Solution:* To transform $\mathcal{A}$ to an equivalent DFA $\mathcal{A}'$, and to find out whether $\mathcal{L}(\mathcal{A}') = \Sigma^*$ (as discussed in the previous tutorial).

**Exercise 8:** Describe an algorithm that for given NFA $\mathcal{A}_1 = (Q_1, \Sigma, \delta_1, I_1, F_1)$ and $\mathcal{A}_2 = (Q_2, \Sigma, \delta_2, I_2, F_2)$ decides if $\mathcal{L}(\mathcal{A}_1) = \mathcal{L}(\mathcal{A}_2)$.

*Solution:* To transform $\mathcal{A}_1$ and $\mathcal{A}_2$ to an equivalent DFAs $\mathcal{A}_1'$ and $\mathcal{A}_2'$, and to find out whether $\mathcal{L}(\mathcal{A}_1') = \mathcal{L}(\mathcal{A}_2')$ (as described in the previous tutorial).

**Exercise 9:** Describe an algorithm that for given GNFA $\mathcal{A}$ constructs an equivalent NFA $\mathcal{A}'$ such that the sets of states of automata $\mathcal{A}$ and $\mathcal{A}'$ are the same.