Tutorial 1

Exercise 1: For each of the following languages, give an example of 5 words belonging to the language, and an example of 5 words that do not belong to the language.

a) $L_1 = \{w \in \{0,1\}^* \mid \text{the length of word } w \text{ is less than } 5\}$
b) $L_2 = \{w \in \{a,b\}^* \mid \text{the number of occurrences of symbol } b \text{ in word } w \text{ is even}\}$
c) $L_3 = \{w \in \{0,1\}^* \mid \text{in } w \text{ is every } 0 \text{ (directly) followed by } 1\}$
d) $L_4 = \{w \in \{0,1\}^* \mid w \text{ begins and ends with the same symbol}\}$
e) $L_5 = \{w \in \{a,b\}^* \mid w \text{ contains as a subword the sequence } abb\}$

Exercise 2: Let us assume $\Sigma = \{a,b\}$ and $n \in \mathbb{N}$.

a) How many words in Σ^* are of length n?

b) How many words in Σ^* are of length at most n?

Exercise 3: Consider the following languages:

$$L_1 = \{w \in \{0,1\}^* \mid \text{in } w \text{ is every } 0 \text{ (directly) followed by } 1\}$$
$$L_2 = \{w \in \{0,1\}^* \mid w = w^R\}$$

a) Enumerate the first 5 words of each of languages L_1, L_2 (the smallest words with respect to order $<_L$).

b) Enumerate the first 5 words of each of languages $\overline{L_1}$, $\overline{L_2}$.

c) Enumerate the first 5 words of language $L_1 \cap L_2$.

d) Enumerate the first 5 words of language $L_1 \cup L_2$.

Exercise 4: For each of the following pairs of languages L_1 and L_2 write down all words from the concatenation of these languages, i.e., from the language $L_1 \cdot L_2$:

a) $L_1 = \{\varepsilon, abb, bba\}$, $L_2 = \{a, b, abba\}$

b) $L_1 = \{0, 001, 111\}$, $L_2 = \{\varepsilon, 01, 0101\}$

c) $L_1 = \{aa, aaaa, aaaaa, aaaaaa\}$, $L_2 = \{aa, aaa\}$

d) $L_1 = \emptyset$, $L_2 = \{011, 1111, 010101\}$

e) $L_1 = \{\varepsilon, a, ba, baa\}$, $L_2 = \{\varepsilon\}$

Exercise 5: Consider languages over the alphabet $\{0, 1\}$. Describe the language of all words in the iteration $\{00, 111\}^*$ and write the first 10 words of the language.

Exercise 6: Consider the following languages:

$$L_1 = \{w \in \{0, 1\}^* \mid |w|_1 \leq 1\}$$
$$L_2 = \{w \in \{0, 1\}^* \mid w = w^R\}$$

Describe the words in the language $L_1 \cap L_2$.

Exercise 7: Determine, which of the following propositions hold generally for all languages. In the case that the given proposition holds for all languages, give a justification why it is the case. In the case that the proposition does not hold generally, give a concrete example of languages for which the given proposition does not hold:

a) $L_1 \cdot L_2 = L_2 \cdot L_1$

b) $L_1 \cdot (L_2 \cdot L_3) = (L_1 \cdot L_2) \cdot L_3$

c) $L_1 \cdot (L_2 \cup L_3) = L_1 \cdot L_2 \cup L_1 \cdot L_3$

d) $L_1 \cdot (L_2 \cap L_3) = L_1 \cdot L_2 \cap L_1 \cdot L_3$

e) $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$

Exercise 8: Describe at least 5 different orders on the set of all words over the alphabet $\{0, 1\}$. For each of them show that it really is an order, i.e., a reflexive, transitive and antisymmetric relation in the case of nonstrict order, or transitive and antisymmetric relation in the case of strict order, and then describe in detail properties of the given order (for example, if it is a total or partial order, if there is some least or greatest element, which elements are minimal or maximal, if there exist infinite decreasing or increasing sequences, etc.)