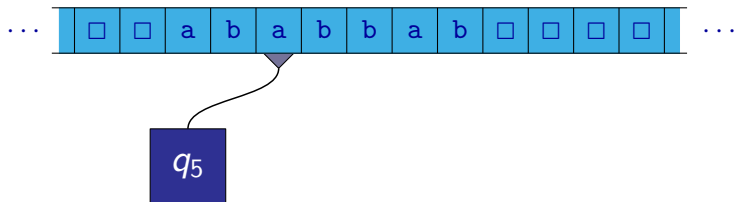


Turing Machines

Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

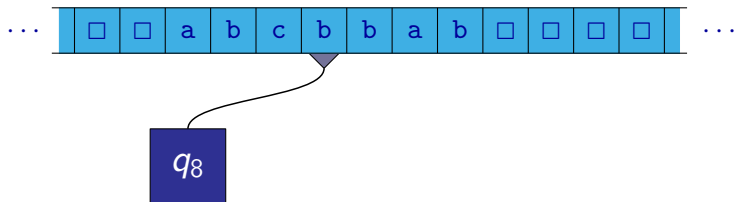
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

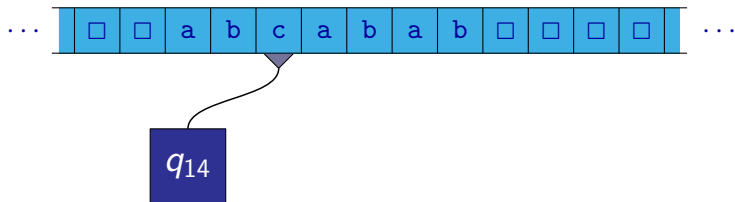
- the head can move in both directions
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- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

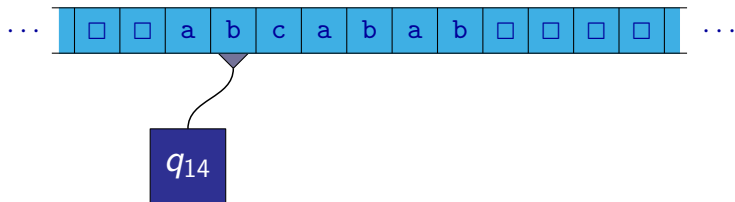
- the head can move in both directions
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- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

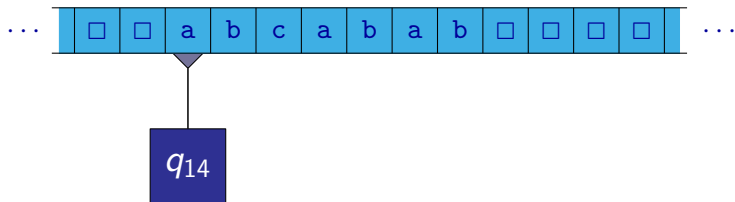
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

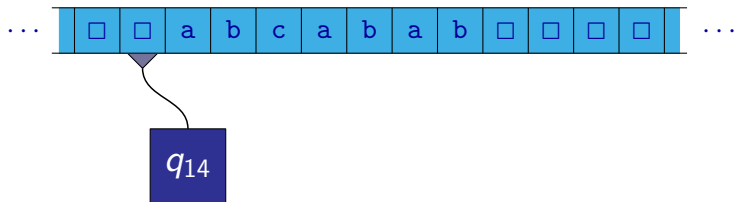
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

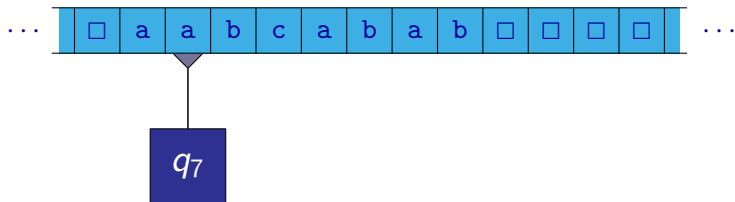
- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Turing Machine

Turing machine — a device similar to a finite automaton with the following differences:

- the head can move in both directions
- it is possible to write on a current position of the head
- the tape is infinite



Alan M. Turing, “On Computable Numbers, with an application to the Entscheidungsproblem”, *Proceedings of the London Mathematical Society*, 42 (1936), pp. 230–265, Erratum: *Ibid.*, 43 (1937), pp. 544–546.

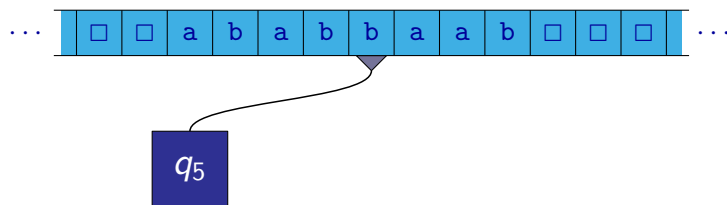
Definition

Formally, **Turing machine** is defined as a tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where:

- Q is a finite non-empty set of **states**
- Γ is a finite (non-empty) set of **tape symbols** (**tape alphabet**)
- $\Sigma \subseteq \Gamma$ is a finite non-empty set of **input symbols** (**input alphabet**)
- $\delta : (Q - F) \times \Gamma \rightarrow Q \times \Gamma \times \{-1, 0, +1\}$ is a **transition function**
- $q_0 \in Q$ is an **initial state**
- $F \subseteq Q$ is a set of **final states**

We assume that $\Gamma - \Sigma$ always contains a special element \square denoting a **blank** symbol.

Configurations of a Turing Machine

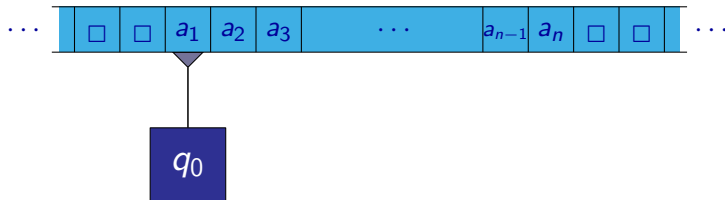


A configuration of a Turing machine is given by:

- a state of its control unit
- a content of the tape
- a position of the head

Configurations of a Turing Machine

A computation of a Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$ over a word $w \in \Sigma^*$, where $w = a_1 a_2 \cdots a_n$, starts in an **initial configuration**:



- the state of the control unit is q_0
- word w is written on the tape, remaining cells of the tape are filled with the blank symbols (\square)
- the head is on the first symbol of the word w (or on symbol \square when $w = \varepsilon$)

One step of a Turing machine:

Let us assume that:

- the state of the control unit is q
- the cell of the tape on the position of the head contains symbol b

Let us say that $\delta(q, b) = (q', b', d)$ where $d \in \{-1, 0, +1\}$.

One step of the Turing machine is performed as follows:

- the state of the control unit is changed to q'
- symbol b' is written on the tape cell on the position of the head instead of b
- The head is moved depending on d :
 - for $d = -1$ the head is moved one cell left
 - for $d = +1$ the head is moved one cell right
 - for $d = 0$ the position of the head is not changed

- A Turing machine performs these steps until a state of its control unit is a state from the set F .
- Those configurations where a state of the control unit belongs to set F are **final configurations**.
- A computation ends in a final configuration.
- A computation of a machine \mathcal{M} over a word w can be infinite.

We often choose the set of final states $F = \{q_{acc}, q_{rej}\}$.

Then we can define for a word $w \in \Sigma^*$ if a given Turing machine accepts it:

- If the state of the control unit after the computation over the word w is q_{acc} , the machine accepts the word w .
- If the state of the control unit after the computation over the word w is q_{rej} , the machine does not accept the word w .
- The computation of the machine over the word w can be infinite. In this case the machine does not accept the word w .

The language $\mathcal{L}(\mathcal{M})$ of a Turing machine \mathcal{M} is the set of all words accepted by \mathcal{M} .

A language $L \subseteq \Sigma^*$ is **accepted** by a Turing machine \mathcal{M} if:

- for each word $w \in \Sigma^*$ it holds that $w \in L$ iff the computation of \mathcal{M} over w ends in final state q_{acc} .

(So computations over words that do not belong to L can end in state q_{rej} or be infinite.)

Language $L \subseteq \Sigma^*$ is **recognized** by a Turing machine \mathcal{M} if:

- for each word $w \in L$ the computation of machine \mathcal{M} over w ends in final state q_{acc} .
- for every word $w \in (\Sigma^* - L)$ the computation of machine \mathcal{M} over w ends in final state q_{rej} .

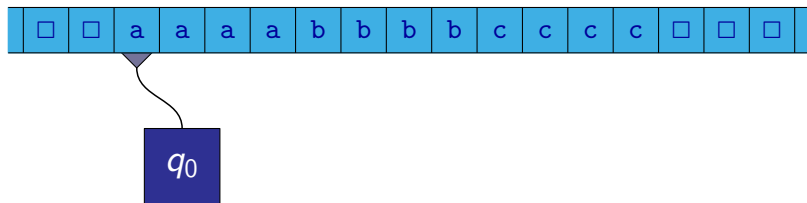
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



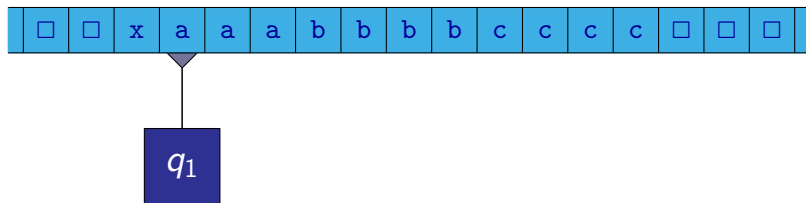
Turing Machine

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$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

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δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



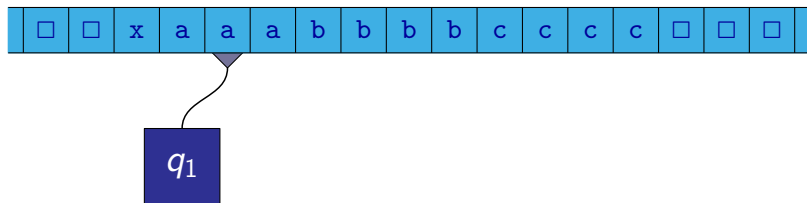
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



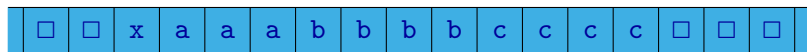
Turing Machine

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$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

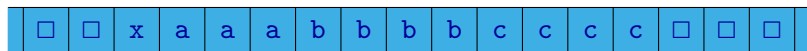
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



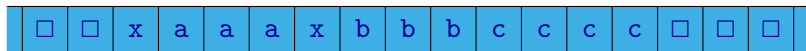
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

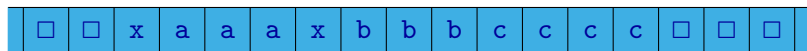
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Language $L = \{a^n b^n c^n \mid n \geq 0\}$

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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

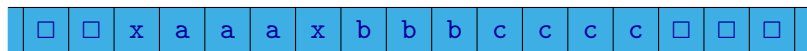
Turing Machine

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δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

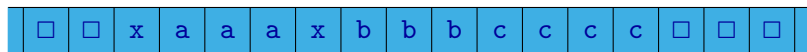
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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

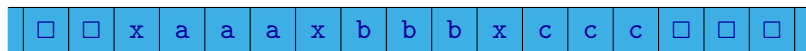
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



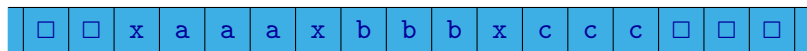
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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_3

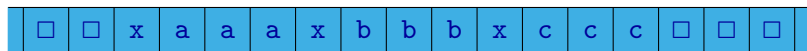
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_3

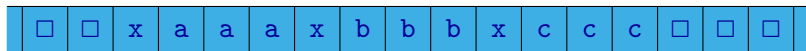
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



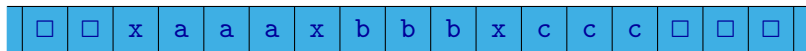
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



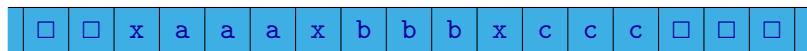
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

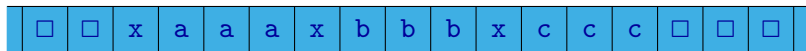
Turing Machine

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$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

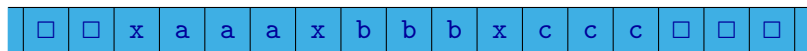
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

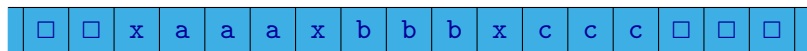
Turing Machine

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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



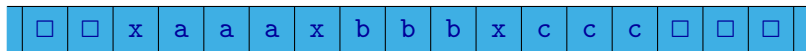
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

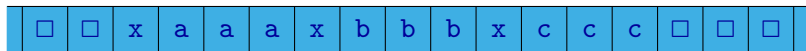
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

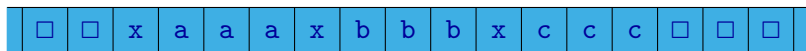
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

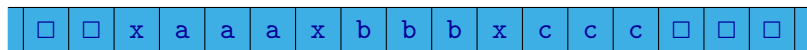
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

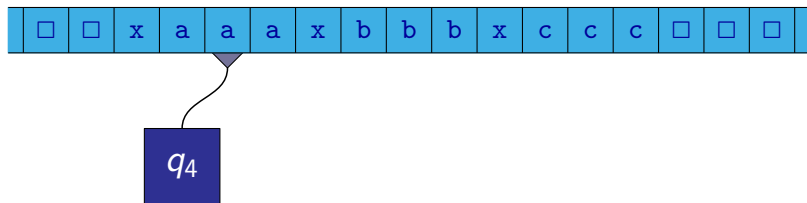
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



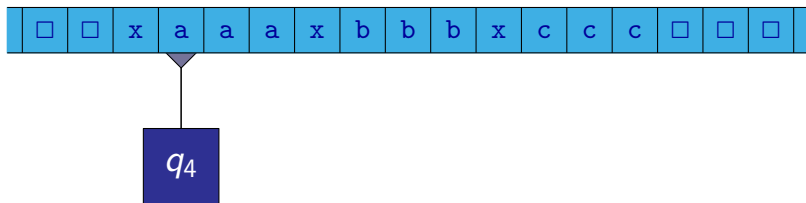
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



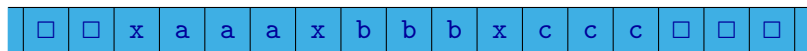
Turing Machine

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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

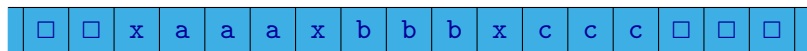
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

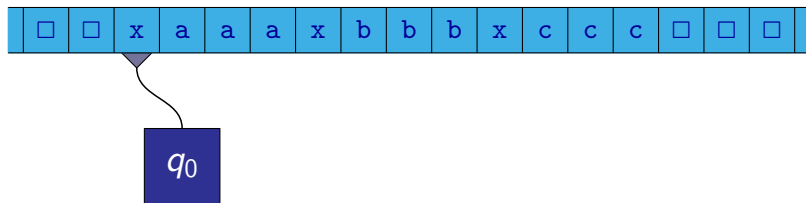
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



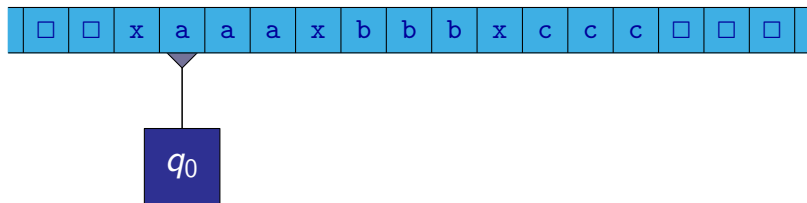
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



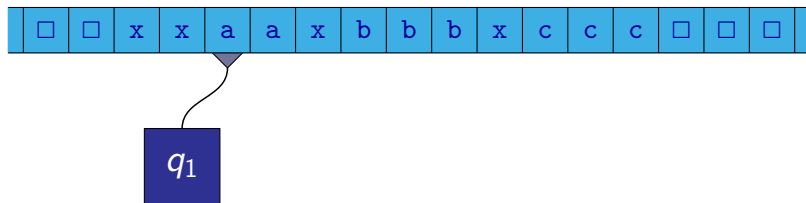
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



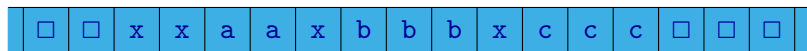
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

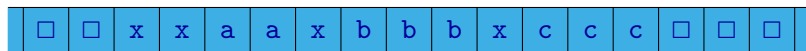
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

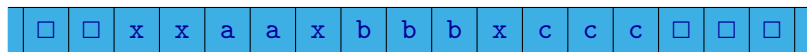
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

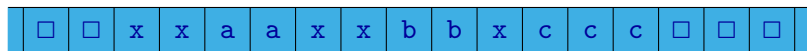
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

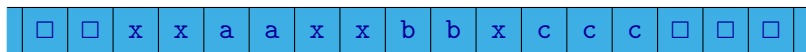
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

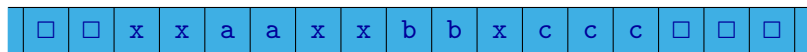
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

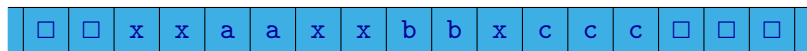
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

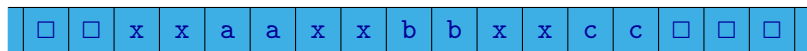
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_3

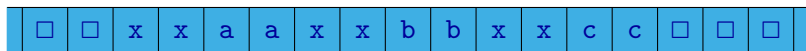
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_3

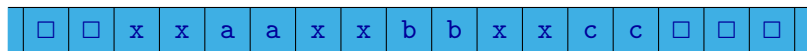
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_3

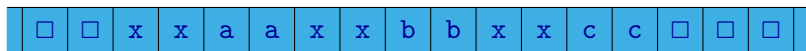
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



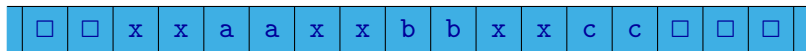
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

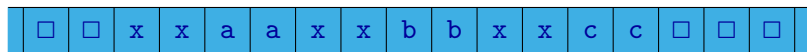
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

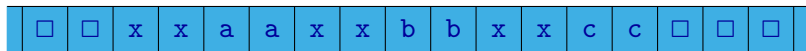
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

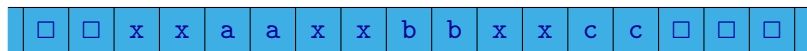
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

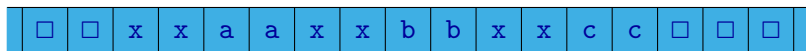
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

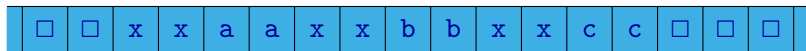
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

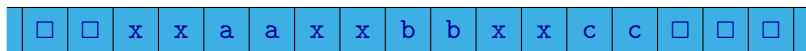
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

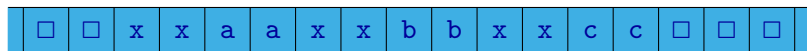
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

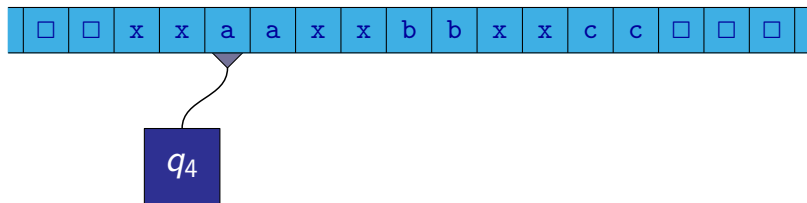
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



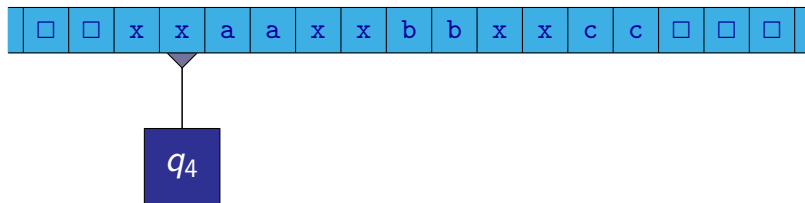
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



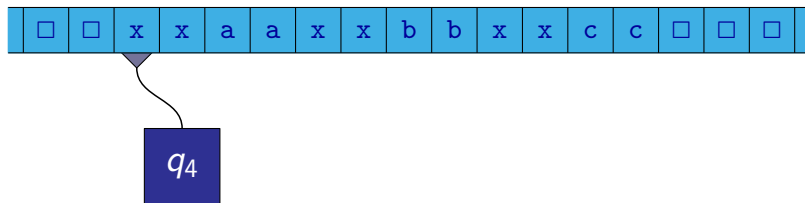
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



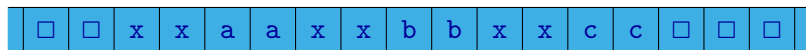
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

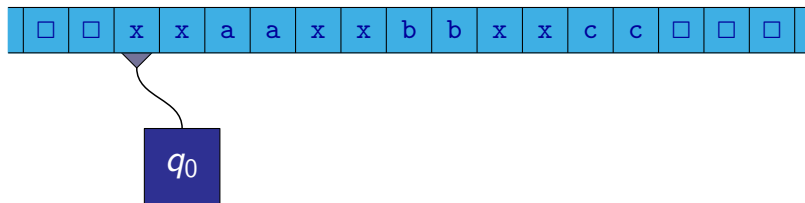
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



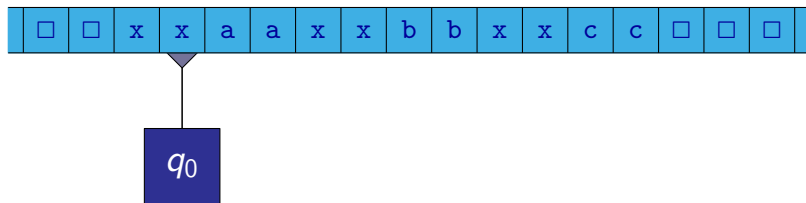
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



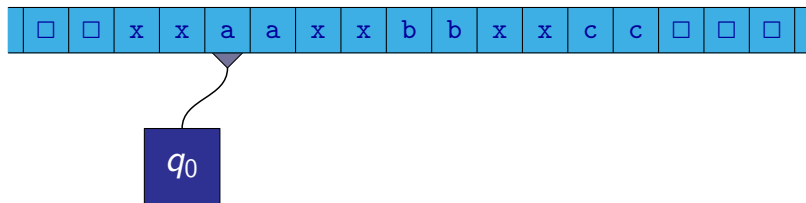
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



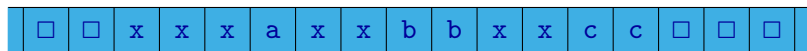
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



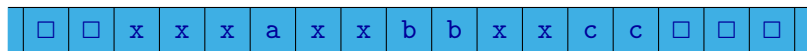
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

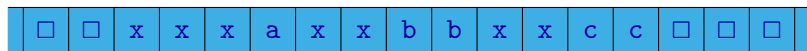
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

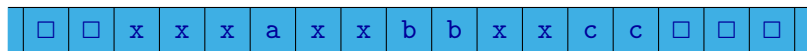
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

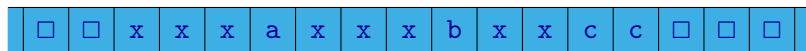
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



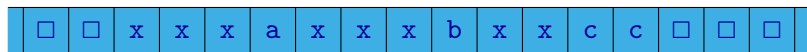
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

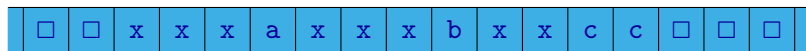
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



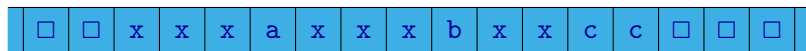
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

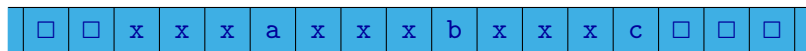
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_3

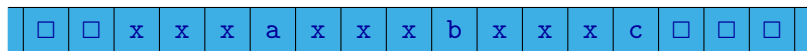
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_3

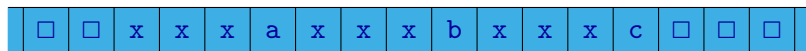
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

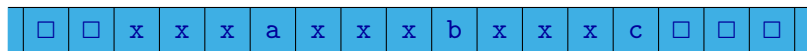
Turing Machine

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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

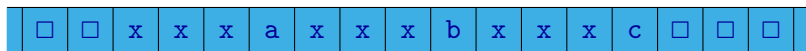
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

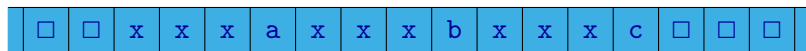
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

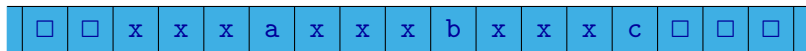
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

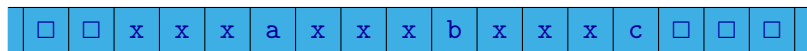
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

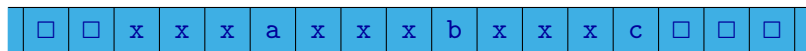
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

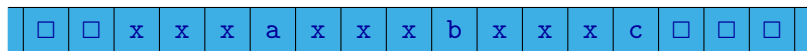
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

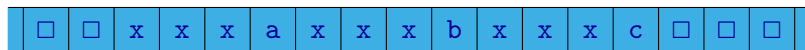
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

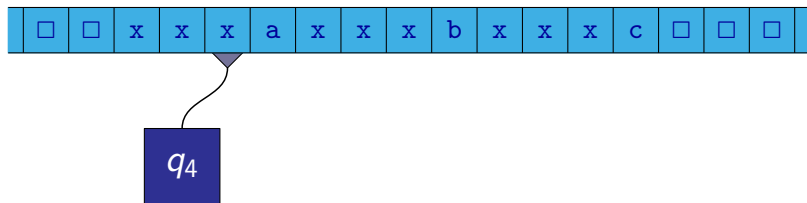
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



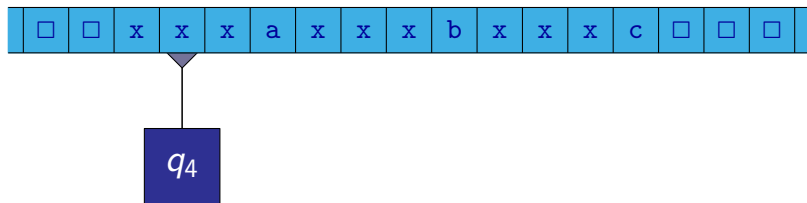
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



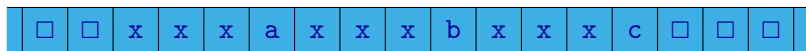
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

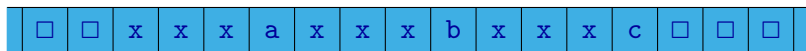
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

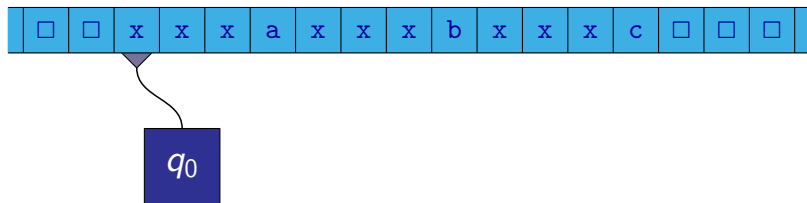
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



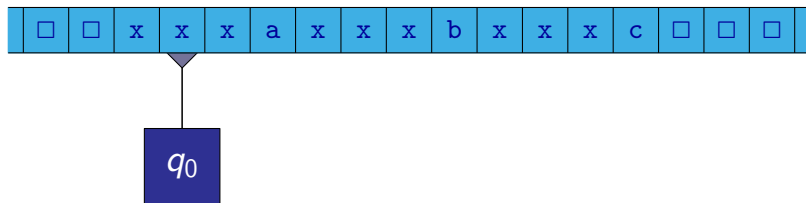
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



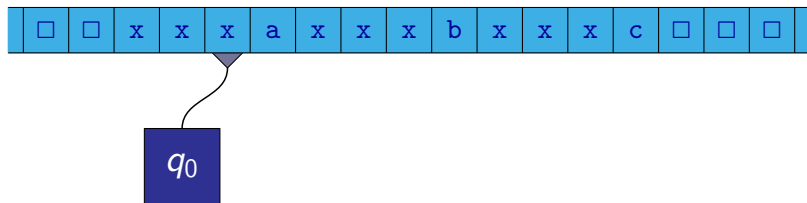
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



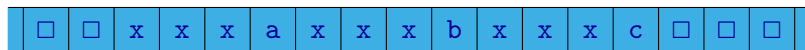
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



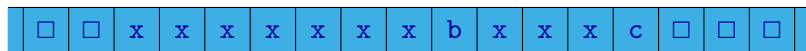
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



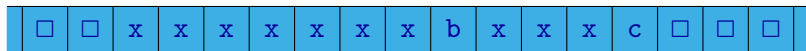
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



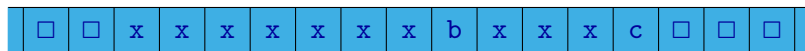
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

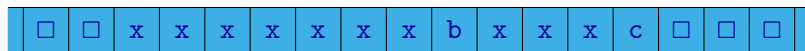
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_1

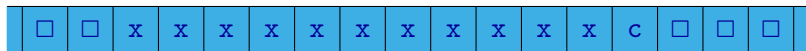
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

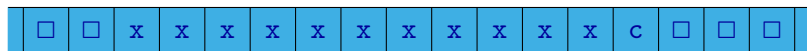
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

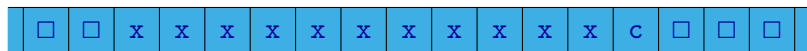
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_2

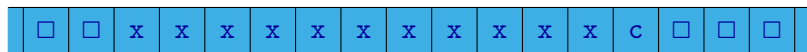
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



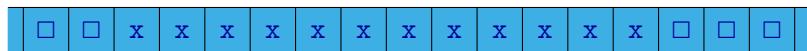
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_3

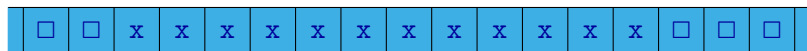
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

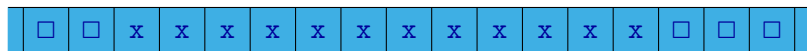
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

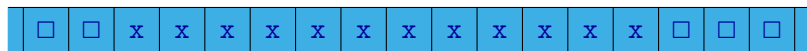
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

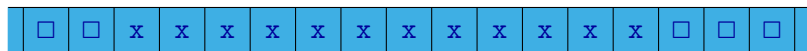
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

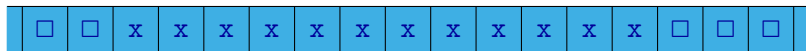
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

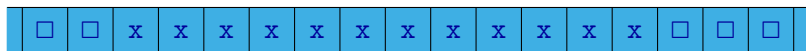
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

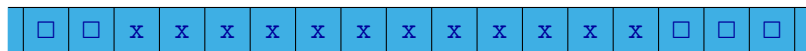
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

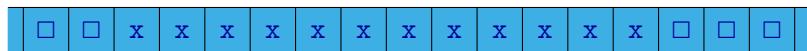
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

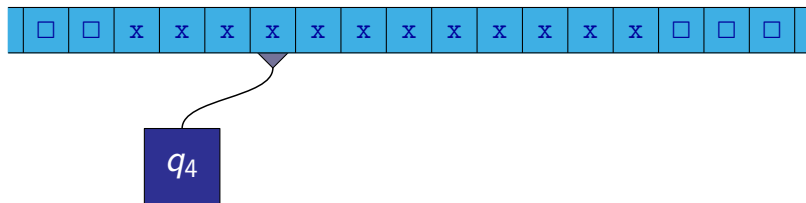
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



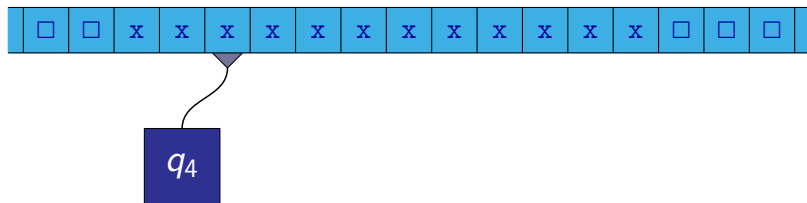
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



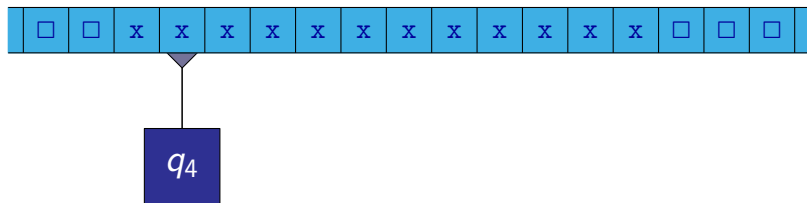
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



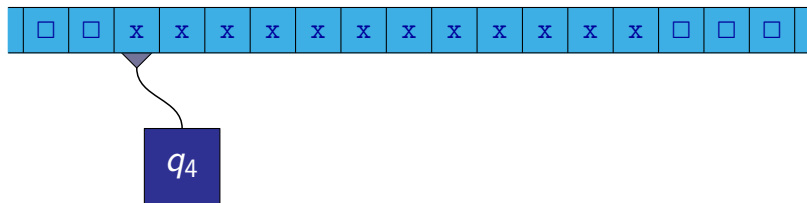
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



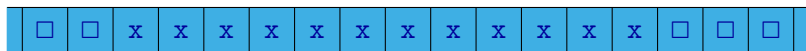
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_4

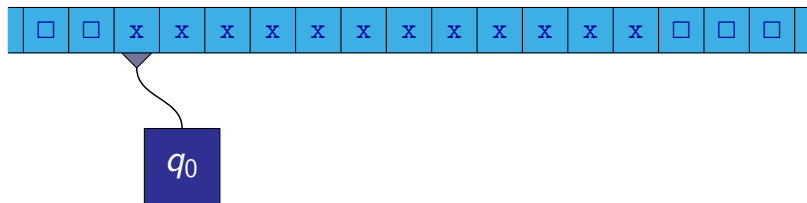
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



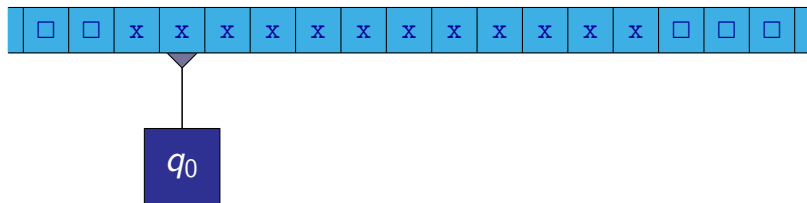
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



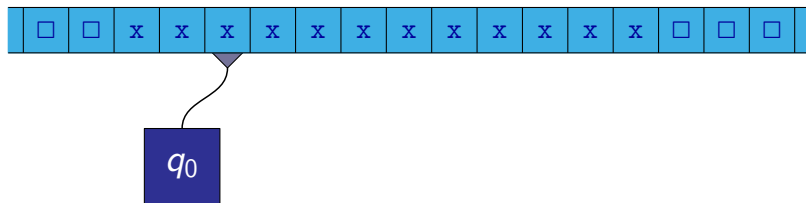
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



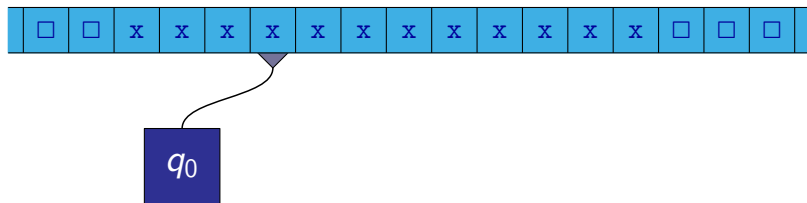
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



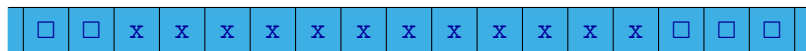
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_0

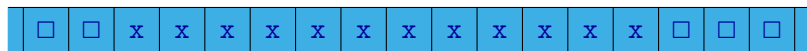
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



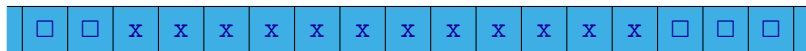
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_0

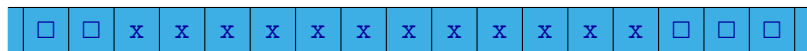
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_0

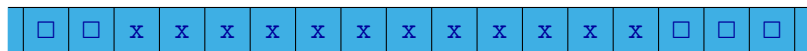
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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



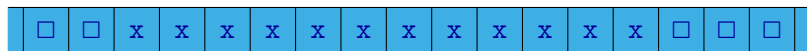
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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



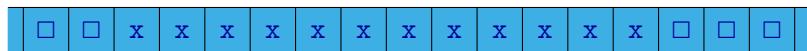
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$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



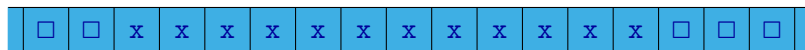
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



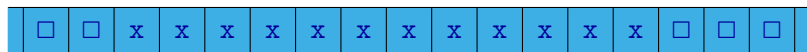
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



q_0

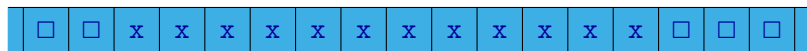
Turing Machine

Language $L = \{a^n b^n c^n \mid n \geq 0\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}$ $F = \{q_{acc}, q_{rej}\}$

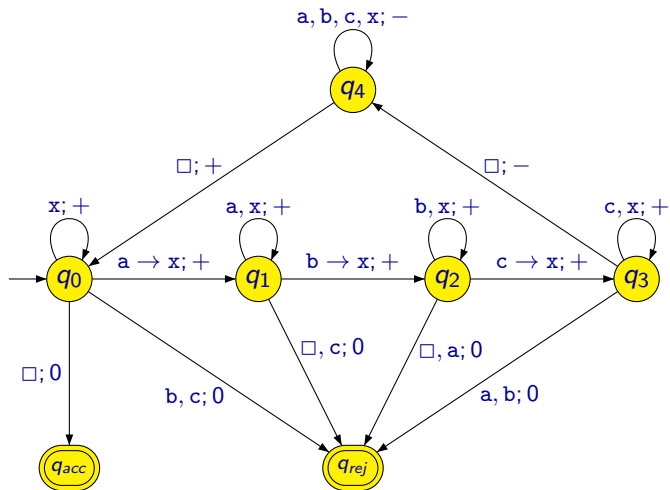
$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, a, b, c, x\}$

δ	\square	a	b	c	x
q_0	$(q_{acc}, \square, 0)$	$(q_1, x, +1)$	$(q_{rej}, b, 0)$	$(q_{rej}, c, 0)$	$(q_0, x, +1)$
q_1	$(q_{rej}, \square, 0)$	$(q_1, a, +1)$	$(q_2, x, +1)$	$(q_{rej}, c, 0)$	$(q_1, x, +1)$
q_2	$(q_{rej}, \square, 0)$	$(q_{rej}, a, 0)$	$(q_2, b, +1)$	$(q_3, x, +1)$	$(q_2, x, +1)$
q_3	$(q_4, \square, -1)$	$(q_{rej}, a, 0)$	$(q_{rej}, b, 0)$	$(q_3, c, +1)$	$(q_3, x, +1)$
q_4	$(q_0, \square, +1)$	$(q_4, a, -1)$	$(q_4, b, -1)$	$(q_4, c, -1)$	$(q_4, x, -1)$



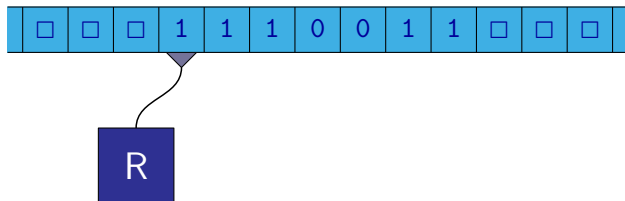
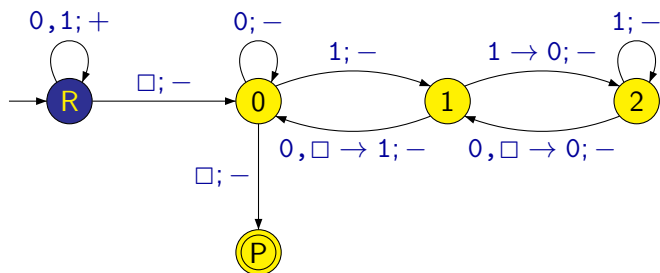
q_{acc}

Turing Machine

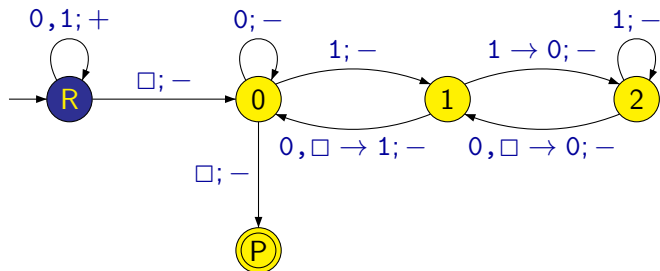


- A Turing machine can give not only answers **YES** or **NO** but it can also compute a function that assigns to each word from Σ^* some other word (from Γ^*).
- A word assigned to a word w is the word that remains on the tape after the computation over the word w when we remove all symbols \square .

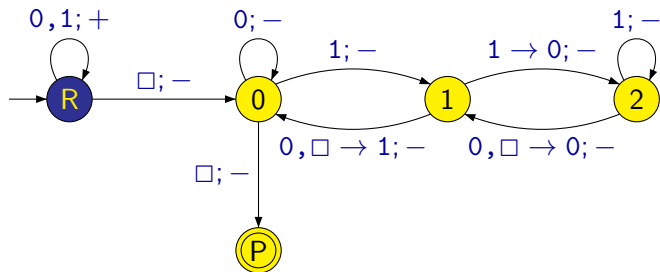
Turing Machine – Multiplication by Three



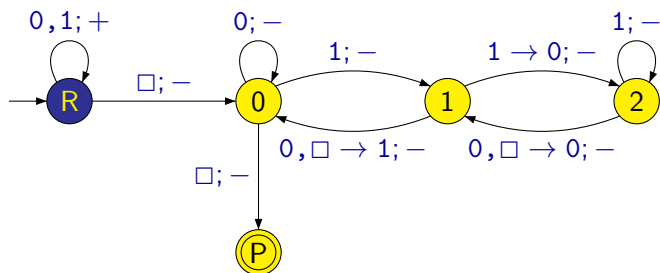
Turing Machine – Multiplication by Three



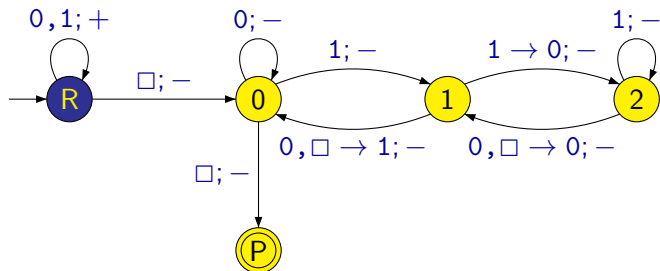
Turing Machine – Multiplication by Three



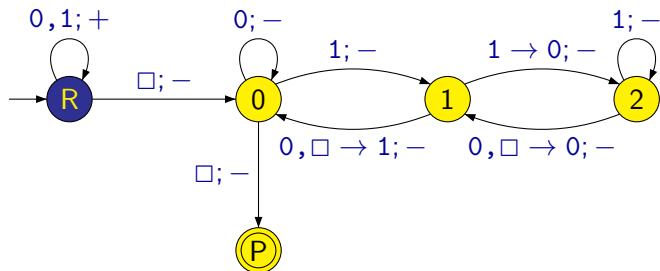
Turing Machine – Multiplication by Three



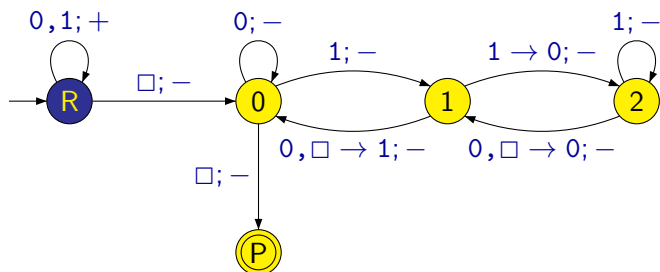
Turing Machine – Multiplication by Three



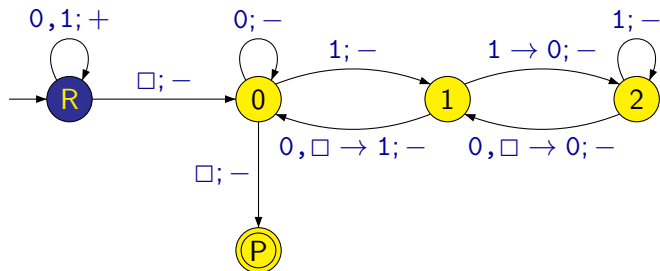
Turing Machine – Multiplication by Three



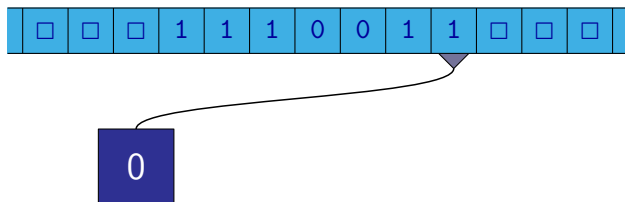
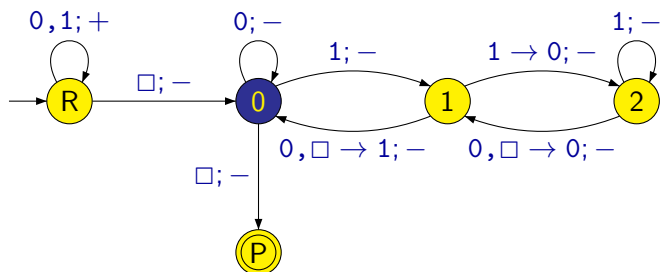
Turing Machine – Multiplication by Three



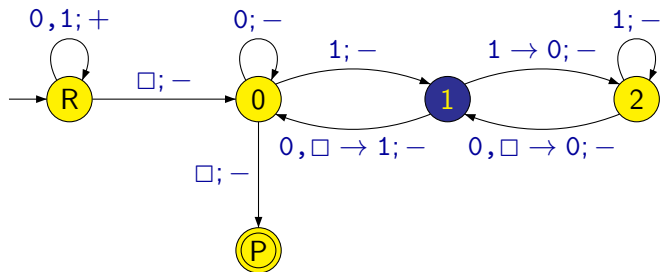
Turing Machine – Multiplication by Three



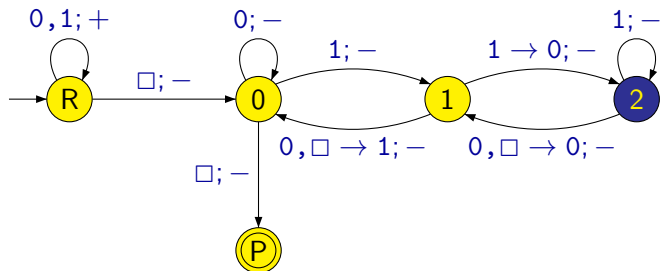
Turing Machine – Multiplication by Three



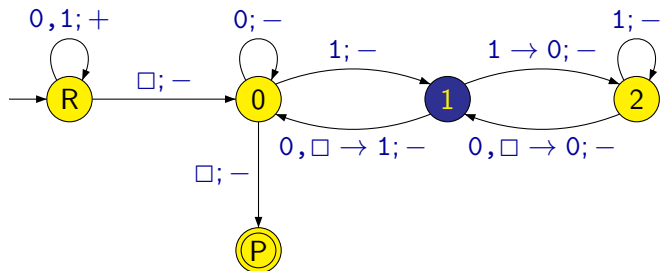
Turing Machine – Multiplication by Three



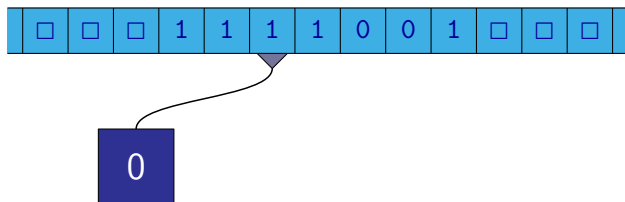
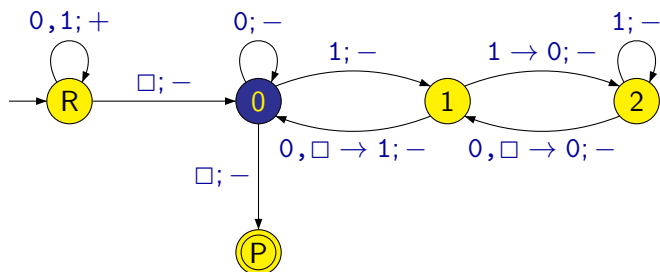
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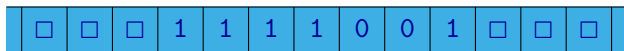
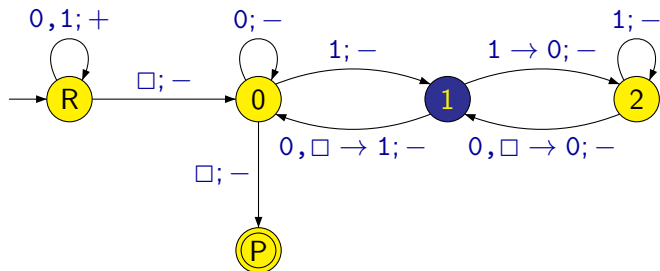
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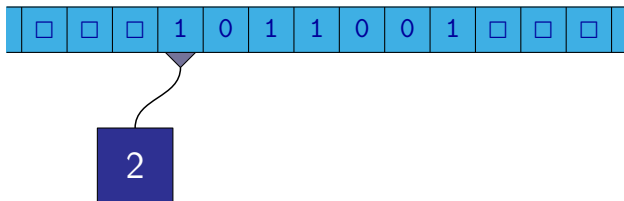
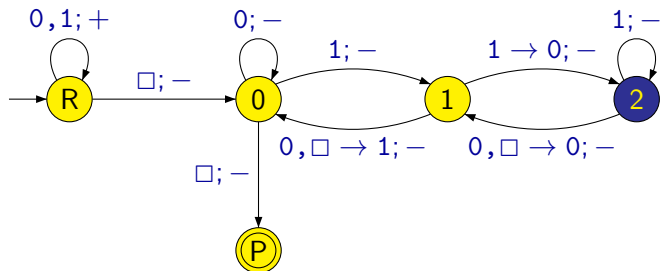
Turing Machine – Multiplication by Three



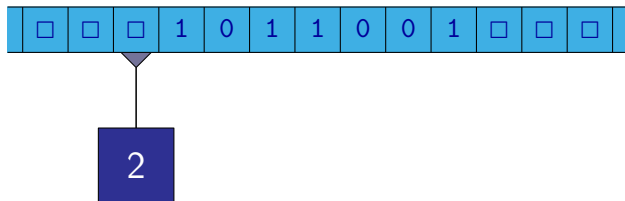
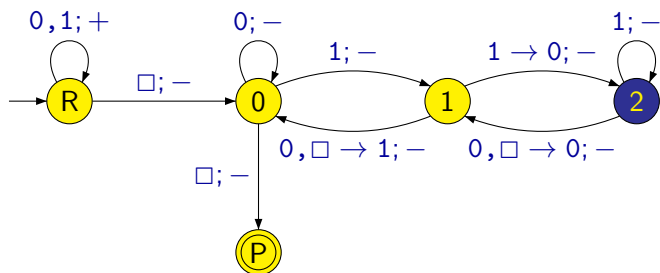
Turing Machine – Multiplication by Three



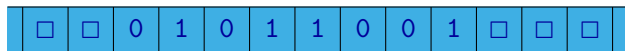
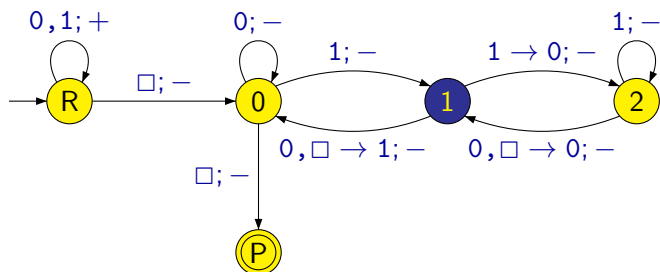
Turing Machine – Multiplication by Three



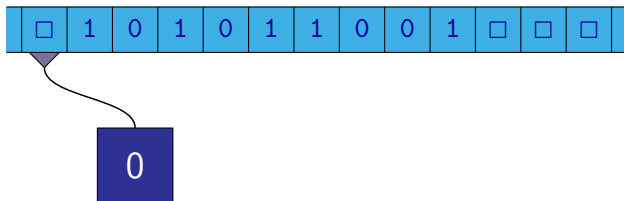
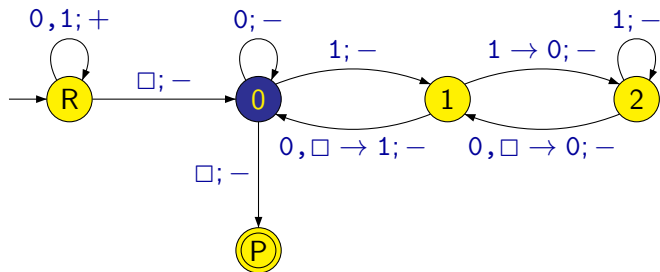
Turing Machine – Multiplication by Three



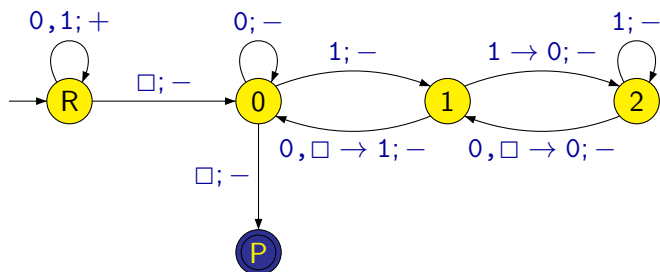
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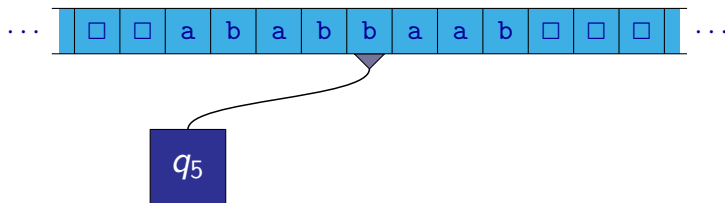
Turing Machine – Multiplication by Three



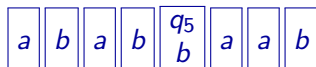
Turing Machine – Multiplication by Three



Configurations of a Turing Machine

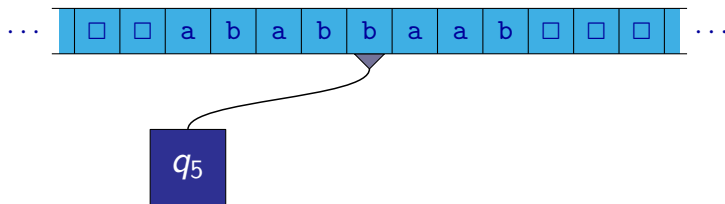


Configurations can be represented as words over alphabet $\Delta = \Gamma \cup (Q \times \Gamma)$:

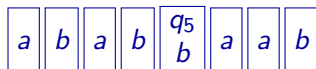


This word always contains exactly one symbol from $(Q \times \Gamma)$ that denotes a state of the control unit and a position of the head.

Configurations of a Turing Machine



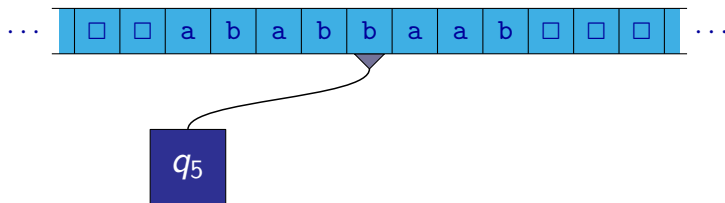
Configurations can be represented as words over alphabet $\Delta = \Gamma \cup (Q \times \Gamma)$:



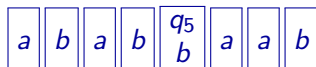
Remark: Symbols from $(Q \times \Gamma)$ can be also written as $\begin{matrix} q \\ a \end{matrix}$ instead of

$\begin{matrix} (q, a) \end{matrix}$.

Configurations of a Turing Machine

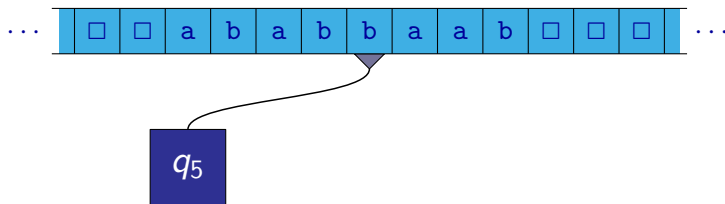


Configurations can be represented as words over alphabet $\Delta = \Gamma \cup (Q \times \Gamma)$:

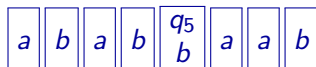



Other symbols (from Γ) represent a content of the tape.

Configurations of a Turing Machine



Configurations can be represented as words over alphabet $\Delta = \Gamma \cup (Q \times \Gamma)$:



Those cells of the tape that are not included in the word contain the symbol .

Configurations of a Turing Machine

Let *Conf* denote the set all configurations of a given machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$.

- If we represent configurations of the Turing machine \mathcal{M} as words over the alphabet $\Delta = \Gamma \cup (Q \times \Gamma)$, the set *Conf* can be identified with the set of those words over alphabet Δ that contain exactly one occurrence of a symbol from $(Q \times \Gamma)$, i.e., with the set of words of the form

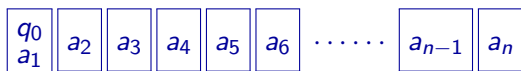
$$u(q, a)v$$

where $q \in Q$, $a \in \Gamma$, $u, v \in \Gamma^*$.

Remark: Alternatively, the configurations could be represented as tuples (u, q, a, v) where $q \in Q$, $a \in \Gamma$, $u, v \in \Gamma^*$.

Configurations of a Turing Machine

The **initial configuration for an input** $w \in \Sigma^*$, where $w = a_1 a_2 \cdots a_n$, is then represented as the word



or (when we use a standard mathematical notation) as:

$$(q_0, a_1) a_2 a_3 \cdots a_{n-1} a_n$$

Remark: If $w = \varepsilon$ then the initial configuration is represented as the word (q_0, \square) .

Computation of a Turing Machine

We want to define formally relation

$$\longrightarrow \subseteq \mathit{Conf} \times \mathit{Conf}$$

containing exactly those pairs of configurations α and α' , for which it holds that the machine \mathcal{M} can go by one step from configuration α to configuration α' , which will be written as

$$\alpha \longrightarrow \alpha'$$

Computation of a Turing Machine

For each configuration α of the form

$$u(q, a)v$$

where $q \in (Q - F)$, $a \in \Gamma$, $u, v \in \Gamma^*$, there exists exactly one configuration α' such that $\alpha \rightarrow \alpha'$.

This configuration α' is determined by the transition function δ .

Let us assume that $\delta(q, a) = (q', a', d)$:

- If $d = 0$ then $\alpha' = u(q', a')v$.
- If $d = -1$:
 - If $u = \varepsilon$ then $\alpha' = (q', \square)a'v$.
 - If $u = u'b$ (where $u' \in \Gamma^*$ and $b \in \Gamma$) then $\alpha' = u'(q', b)a'v$.
- If $d = +1$:
 - If $v = \varepsilon$ then $\alpha' = ua'(q', \square)$.
 - If $v = bv'$ (where $b \in \Gamma$ and $v' \in \Gamma^*$) then $\alpha' = ua'(q', b)v'$.

Remark:

- The tape represents a “memory” with infinite capacity where the Turing machine can store information.
- This memory is infinite potentially — every configuration can be viewed as a finite object (represented as a finite word over the alphabet Δ , as described above).

Computation of a Turing Machine

- We can imagine that the machine does not work with an infinite tape but rather that in every step there is only a finite number of cells of the tape.
- When the head occurs on the left or the right boundary of the tape and it is necessary to move the head out of the currently existing cells of the tape, a new cell with the symbol \square is automatically added to the given boundary of the tape.
- Steps of this kind can be viewed as a certain type of “memory allocation” .
- There is an idealization in that we assume that this “allocation” of cells of the tape never fails.

Computation of a Turing Machine

Final configurations are configurations of the form

$$u(q, a)v$$

where $q \in F$, $a \in \Gamma$, $u, v \in \Gamma^*$.

For a final configuration α , there is no configuration α' such that $\alpha \longrightarrow \alpha'$.

A computation of a Turing machine \mathcal{M} over a word w is then defined as usual as a finite or infinite sequence of configurations

$$\alpha_0 \longrightarrow \alpha_1 \longrightarrow \alpha_2 \longrightarrow \alpha_3 \longrightarrow \dots$$

where α_0 is the initial configuration of the machine \mathcal{M} on the word w .

Computation of a Turing Machine

Later, when we will study the time and space complexity, the following two values will play an important role:

- the **number of steps** that the given machine performs in the computation over a given word
 - this value represents the running time of the computation
- the **number of cells of the tape** visited by the given machine during the computation over a given word
 - this value represents the amount of the used memory

Computation of a Turing Machine

Formally, these notions can be defined as follows
(we assume a given Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$):

- Function

$$time_{\mathcal{M}} : \Sigma^* \rightarrow \mathbb{N} \cup \{\infty\}$$

For $w \in \Sigma^*$, the value $time_{\mathcal{M}}(w)$ specifies the number of steps the machine \mathcal{M} performs in a computation over the word w .

I.e., if this computation is finite and looks as follows

$$\alpha_0 \longrightarrow \alpha_1 \longrightarrow \alpha_2 \longrightarrow \alpha_3 \longrightarrow \cdots \longrightarrow \alpha_{t-1} \longrightarrow \alpha_t$$

where α_t is a final configuration, then $time_{\mathcal{M}}(w) = t$.

In the case of an infinite computation

$$\alpha_0 \longrightarrow \alpha_1 \longrightarrow \alpha_2 \longrightarrow \alpha_3 \longrightarrow \cdots$$

we have $time_{\mathcal{M}}(w) = \infty$.

- Function

$$space_{\mathcal{M}} : \Sigma^* \rightarrow \mathbb{N} \cup \{\infty\}$$

For $w \in \Sigma^*$, the value $space_{\mathcal{M}}(w)$ specifies the number of cells that the machine \mathcal{M} visits during a computation over an the input w .

Remark: It is obvious that in the case of a finite computation also the value $space_{\mathcal{M}}(w)$ is finite.

In the case of an infinite computation, the value $space_{\mathcal{M}}(w)$ can be finite or infinite.

Nondeterministic Turing Machines

We can also consider **nondeterministic Turing machines** where for every state q and symbol b the transition function $\delta(q, b)$ specifies several different triples (q', b', d) .

The machine can choose any of them.

The machine accepts a word w iff it has at least one computation where w is accepted.

Remark: For every nondeterministic Turing machine, there can be constructed an equivalent deterministic Turing machine.

Nondeterministic Turing Machines

Formally, the only difference in the definition of a deterministic and a nondeterministic Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$ is the definition of the transition function δ :

- **Deterministic** Turing machine:

$$\delta: (Q - F) \times \Gamma \rightarrow Q \times \Gamma \times \{-1, 0, +1\}$$

- **Nondeterministic** Turing machine:

$$\delta: (Q - F) \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{-1, 0, +1\})$$

Remark: For nondeterministic Turing machines, it makes a little sense to consider other set of final state than $F = \{q_{acc}, q_{rej}\}$.

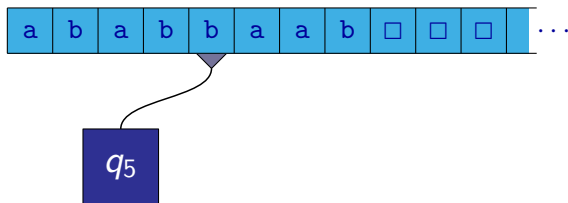
Variants of Turing Machines

- The definition of Turing machine given before is just one of many variants.
- Here we give several examples of differences between different variants of Turing machines.
- Almost all these variants of Turing machines are able to accept or recognize the same languages and to compute the same functions.
- There can be (but need not be) big differences between variants with respect to their running time and an amount of used memory.
- All these variants can be considered in a deterministic and a nondeterministic version.

Variants of Turing Machines

One-sided or **two-sided** infinite tape:

- In the previous definition, we have considered a tape that is infinite in both directions — to the left and to the right.
- Instead, it is sometimes considered a tape that is infinite only to the right.

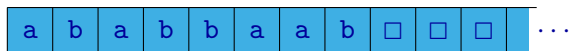


Variants of Turing Machines

It is necessary to define what should happen when the head is on the leftmost cell of the tape and, according to the transition function, it should move to the left.

Two most common possibilities:

- An “error” occurs and the computation is (unsuccessfully) ended:

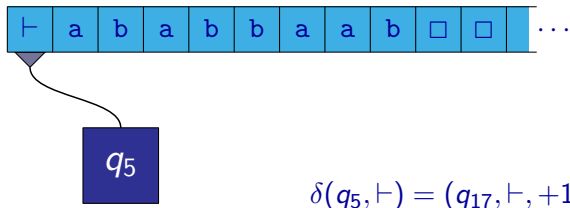


$$\delta(q_5, a) = (q_{13}, b, -1)$$

Variants of Turing Machines

- The left end of the tape contains a “marker” represented by a special symbol $\vdash \in (\Gamma - \Sigma)$.

This marker can not be overwritten and a move to the left is forbidden on this symbol, i.e., for each $q \in Q$ it holds that if $\delta(q, \vdash) = (q', b, d)$ then $b = \vdash$ a $d \in \{0, +1\}$.



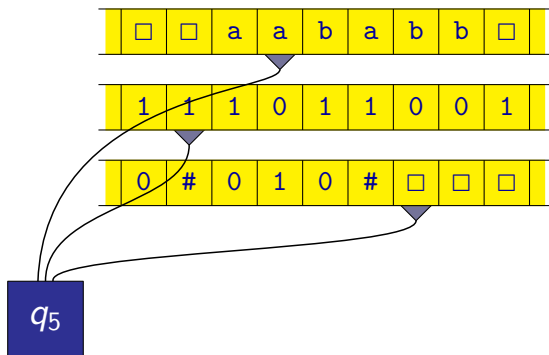
Remark: The possibility that a computation can end unsuccessfully because of an error where it is no possible to continue from the given configuration is quite common also for other types of machines we will consider.

Generally, the following possibilities can happen in a computation:

- The computation ends successfully in a final configuration that corresponds to a correct halting.
- The computation is stuck in a configuration that is not final but it is not possible to continue there — this is considered as a computation ending with an error.
- The computation never halts.

Variants of Turing Machines

Multitape Turing machines are often considered.



Variants of Turing Machines

In the case of a multitape machines:

- Each of k tapes has its own alphabet, i.e., we have tape alphabets $\Gamma_1, \Gamma_2, \dots, \Gamma_k$.
- The transition function δ is of the type

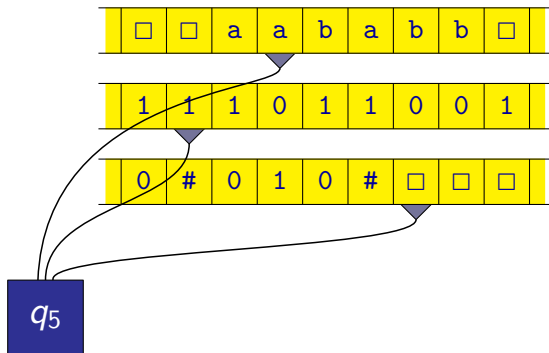
$$(Q - F) \times \Gamma_1 \times \dots \times \Gamma_k \rightarrow Q \times \Gamma_1 \times \{-1, 0, +1\} \times \dots \times \Gamma_k \times \{-1, 0, +1\}$$

Example:

$$\delta(q_5, a, 1, \square) = (q_{12}, a, -1, x, 0, 1, +1)$$

Variants of Turing Machines

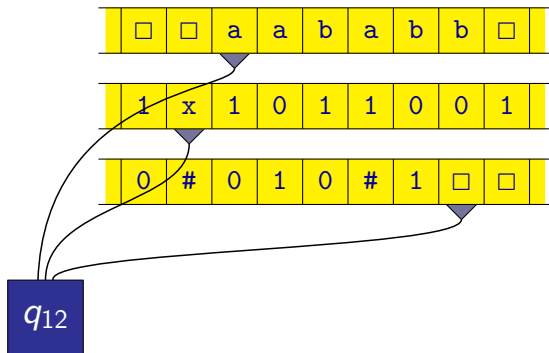
Example:



$$\delta(q_5, a, 1, \square) = (q_{12}, a, -1, x, 0, 1, +1)$$

Variants of Turing Machines

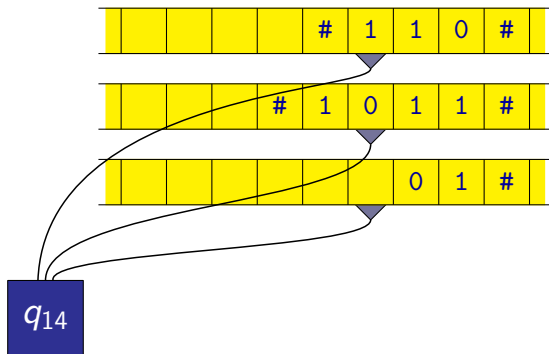
Example:



$$\delta(q_5, a, 1, \square) = (q_{12}, a, -1, x, 0, 1, +1)$$

Variants of Turing Machines

Example: A machine that gets as an input two natural numbers written in binary and separated by symbols # (e.g., number 6 and 11 will be written as “#110#” a “#1011#”).

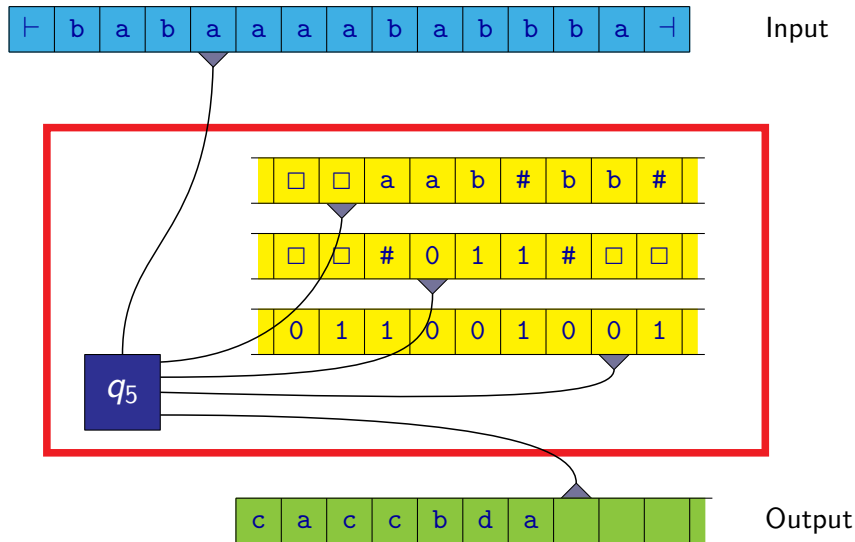


Variants of Turing Machines

Multitape machines often use one of its tapes as an input tape and one of its tapes as an output tape. Other tapes are used as working tapes:

- **Input tape** — it contains an input word, the machine can not write on it (it is read-only), it is not infinite
- **Working tapes** — the machine can read from them and write on them (they are read/write), at the beginning of a computation they are empty (they contain only symbols \square)
- **Output tape** — the machine can only write on it (it is write-only), it can not read from it, it is empty at the beginning of a computation, the head can move only from the left to the right

Variants of Turing Machines



Variants of Turing Machines

If a machine has a special separate input tape (read-only) and special separate output tape (write-only), these tapes are typically not counted in the total memory needed during a computation.

In this case, the value $space_M(w)$ specifies the total number of cells on the working tapes visited during a computation.

Remark: The amount of used memory defined in this way makes sense specifically in those cases where the number of cells on working tapes is considerably smaller than the size of an input and output.

Variants of Turing Machines

If a machine has a special separate input tape (which is read-only), the following two variants are typically used:

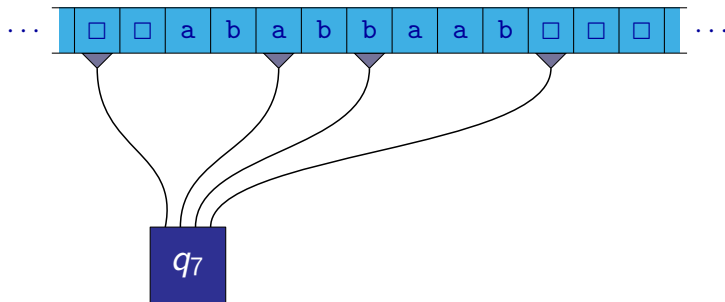
- The head on this tape can move to the left and to the right.
In this case, an input word $w \in \Sigma^*$ is bounded from the left and from the right using “endmarkers”, i.e., special symbols $\vdash, \dashv \in (\Gamma - \Sigma)$.
- The head can move only from left to right.

Remark: The variant with possible movement in both directions and endmarkers is more common.

If it is not specified otherwise, we will consider this variant.

Variants of Turing Machines

Instead of several tapes, we can consider **several heads** on one tape:



In the variant with several heads on one tape, it is necessary to specify:

- If there can be more than one head in the same time on one tape cell.
- If this is the case, what is the behaviour of the machine if several heads occurring on the same cell want to write different symbols on this cell.
- Whether the given machine can detect the situation when several head are on the same cell.

Remark: Of course, in general we can consider machines with several tapes where each of these tapes is equipped with several heads.

Variants of Turing Machines

Consider a machine with several tapes and with arbitrary number of heads on each tape.

Instead of describing a transition function that works with all heads in each step, we can alternatively describe the behaviour of the machine by a **program** consisting of simpler instructions of the following types:

- to move a given head by one cell to the left
- to move a given head by one cell to the right
- to write a specified symbol on the given position of a specified head on a tape
- to read one symbol from a position of a given head and to branch the program according this symbol (i.e., to go to different states of the control unit)

Variants of Turing Machines

So far we considered only **linear** (one-dimensional) tapes.

Instead, the memory with cells (where every cell contains one symbol from some alphabet) can have some other structure.

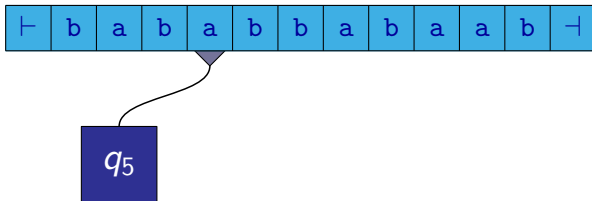
For example:

- two-dimensional **square grid**
— a movement of a head into four directions: left, right, up, down
- d -dimensional memory for some $d = 3, 4, \dots$
(three-dimensional, four-dimensional, etc.)
- a memory organized in a form of an (infinite) tree
- ...

Linear Bounded Automaton

Linear bounded automaton (LBA):

- A nondeterministic Turing machine that can use only the part of the tape where its input word is written.
- Cells of the tape, which at the beginning contain symbols of an input word, can be arbitrarily overwritten during a computation.
- Left and right endmarkers around the word. These endmarkers can not be overwritten.
- It is not possible to move the head to the left of the left endmarker and to the right of the right endmarker.



Linear Bounded Automaton

- Linear bounded automata can be considered in both **deterministic** and **nondeterministic** version.
- The nondeterministic version is considered as the default (i.e., if it is not specified otherwise).
- The question whether every language that can be recognized by a nondeterministic LBA can be also also recognized by a deterministic LBA is an open problem.

Remark: From the point of view of languages that they are able to accept or recognize and from the point view of functions that they can compute, linear bounded automata are considerably weaker than Turing machines that can use memory of unbounded size (in the form of an infinite tape).

Computation of a Turing machine as Data

Consider now arbitrary variants of Turing machines — i.e., with arbitrary number of tapes, heads, etc.

It is not difficult to realise that it is possible to work with configurations of a given machine \mathcal{M} as with data.

For example, configurations of a Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, F)$ can be encoded as words over alphabet $\Delta = \Gamma \cup (Q \times \Gamma)$:

- In a representation of a configuration, there is always exactly one symbol from the set $(Q \times \Gamma)$ — it represents the control state and the position of the head.
- Remaining symbols (from set Γ) represent the content of the tape.

Similarly, we can propose some particular way how to encode configurations in a form of data (e.g., as words over some alphabet) for other types of Turing machines.

Computation of a Turing machine as Data

Also a **computation** (resp. a description of this computation) of a given machine \mathcal{M} on some input w can be viewed as a certain sort of data.

A computation can be represented as a sequence of configurations separated by symbol $\# \notin \Delta$.

In a computation

$$\alpha_0 \longrightarrow \alpha_1 \longrightarrow \alpha_2 \longrightarrow \alpha_3 \longrightarrow \cdots \longrightarrow \alpha_{t-1} \longrightarrow \alpha_t$$

two consecutive configurations α_{j-1} and α_j , i.e., such that

$$\alpha_{j-1} \longrightarrow \alpha_j$$

differ only in limited number of positions.

	0	1	2	3	n $n+1$			j							
α_0	<input type="checkbox"/>	q_0 a_1	a_2	a_3		a_n	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>				<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
α_1															
α_2															
α_3															
α_i															
α_{t-1}															
α_t										q_{acc} x					

Computation of a Turing machine as Data

A description of arbitrary Turing machine \mathcal{M} (or other computation model) can be represented in a form of data — e.g., a word over some alphabet.

Let $Code(\mathcal{M})$ denote such representation of machine \mathcal{M} (in some particular format).

- A representation $Code(\mathcal{M})$ contains information about states of the control unit, a transition functions, etc.
- $Code(\mathcal{M})$ can be viewed as a code of a program.

Universal Turing Machine

A **universal Turing machine** \mathcal{U} is a machine that when obtains a word $Code(\mathcal{M})$ and word $w \in \Sigma^*$ (where Σ is the input alphabet of the machine \mathcal{M}), starts to simulate a computation of the machine \mathcal{M} over input w .

(The machine \mathcal{U} can obtain the input $Code(\mathcal{M})$ and w for example as a word $Code(\mathcal{M})\#w$.)

So the universal Turing machine is able to perform a computation of any other Turing machine (whose description it obtains as a part of its input).

Remark: This corresponds to a situation where we have:

- hardware of a computer (machine \mathcal{U}), which is able to execute an arbitrary algorithm
- code of a program ($Code(\mathcal{M})$) running on this computer
- input data for this program (word w)