Tutorial 7

Exercise 1: Construct a Turing machine $\mathcal{M}$ deciding membership of a word in language $L = \{ w \in \{a, b\}^* \mid w = w^R \}$, i.e., for an input $w \in \{a, b\}^*$, $\mathcal{M}$ gives answer YES when $w \in L$, and answer NO otherwise.

Exercise 2: Construct a Turing machine that erases from a given word over alphabet $\{a, b\}$ the longest possible sequences of a's of the same length from its start and its end. (From word '$aaababaa$' it makes '$abab$', and from '$aaabab$' it does not erase anything. From 'aaa' it makes ε.)

Exercise 3: Construct a Turing machine that from a given word over alphabet $\{a, b, c\}$ removes all occurrences of symbol a. We assume that TM starts its computation on the first symbol of the word on the left.

Exercise 4: Construct a Turing machine that divides a given number written in binary by three.
Hint: Recall the standard elementary school algorithm for division of numbers and proceed directly according to this algorithm.

Exercise 5: Construct a one-tape Turing machine working with the (tape) alphabet $\{a, b, c, \square\}$ that performs the following computation:
At the beginning, some word $w \in \{a, b\}^*$ is written on the tape and the rest of the tape is filled with $\square$. The head of the machine is on the first symbol of w. Your Turing machine must always halt and after the ending of its computation there must be the word $\underbrace{c \ldots c}_{k}$ written on the tape where k is the number of changes between a and b (in both directions, i.e., you count $\ldots ab \ldots$ and also $\ldots ba \ldots$) in the given word w. The rest of the tape must be filled with $\square$.

Hint: Roughly speaking, a computation of your machine must count all changes form a to b and from b to a in a word w, and then "write" it as the number of c's. For example, for input aaa the result is ε, for $aaab$ the result is c, for $ababa$ the result is $cccc$, and for $aabbbbaabbbba$ also $cccc$.

Exercise 6: Describe how a Turing machine with arbitrary tape alphabet Γ can be simulated by a Turing machine with tape alphabet $\Gamma = \{0, 1, \square\}$ that never writes symbol $\square$ on the tape.

Exercise 7: Describe how a Turing machine with one-side infinite tape can simulate a computation of a Turing machine with a tape infinite on both sides.

Exercise 8: Construct a Turing machine with two tapes deciding membership of a word in language $L = \{ w \in \{a, b\}^* \mid w = w^R \}$.

Exercise 9: Describe how a Turing machine with two tapes can be simulated by a Turing machine with one tape (and one head). Generalize your solution to arbitrary finite number of tapes.