Tutorial 6

Exercise 1: Construct a context-free grammar generating the set of all well-formed formulas of the propositional logic. Consider the set $At = \{x_0, x_1, x_2, \ldots\}$ as the set of atomic propositions, where individual variables can be written as `x0`, `x1`, `x2`, ...

a) Find out if the grammar you have constructed is unambiguous.

b) If the grammar is ambiguous then modify it to be unambiguous.

c) Modify your grammar in such a way, which ensures that a structure of a derivation tree for an arbitrary derivation in the grammar reflects the "real" priority of logical connectives, i.e.. $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ (from the highest to the lowest).

Exercise 2: Construct a reduced grammar equivalent to the following grammar:

$$\begin{array}{lcl} S & \longrightarrow & aBC \mid aCa \mid bBCa \\ B & \longrightarrow & bBa \mid bab \mid SS \\ C & \longrightarrow & BS \mid aCaa \mid bSSc \end{array}$$

Exercise 3: Construct a grammar without ε-rules generating language $L(\mathcal{G}) - \{\varepsilon\}$ for the following grammar $\mathcal{G}$:

$$\begin{array}{lcl} S & \longrightarrow & AB \mid \varepsilon \\ A & \longrightarrow & aAAb \mid BS \mid CA \\ B & \longrightarrow & BbA \mid CaC \mid \varepsilon \\ C & \longrightarrow & aBB \mid bS \end{array}$$

Exercise 4: Transform the following grammar to Chomsky normal form:

$$\begin{array}{lcl} S & \longrightarrow & (E) \mid E \\ E & \longrightarrow & F + F \mid F \times F \\ F & \longrightarrow & a \mid S \end{array}$$

Exercise 5: Construct a pushdown automaton accepting the following language:

$$L = \{w \in \{a, b\}^* \mid |w| \geq 1 \text{ and } |w|_a = |w|_b\}$$

If you succeeded, then try to construct a deterministic pushdown automaton for the language L. Is it possible? If not, why?

Then try to construct a deterministic pushdown automaton for the language $L \cdot \{\$\}$ (over alphabet $\{a, b, \$\}$).

Exercise 6: Use a general construction that constructs a (one-state) pushdown automaton for a given context-free grammar to constuct a pushdown automaton accepting language generated by the following grammar:

$$\begin{aligned} A &\longrightarrow A + B \mid B \\ B &\longrightarrow B * C \mid C \\ C &\longrightarrow (A) \mid a \end{aligned}$$

Simulate an accepting computation of the constucted automaton on word $a * (a + a)$.

Exercise 7: Show that the class of context-free languages is closed with respect to the intersection with a regular language, i.e., that for every context-free language L and a regular language R, the language $L \cap R$ is context-free.

Exercise 8: Show that (nondeterministic) pushdown automata accepting by empty stack and accepting by final state have the same expressive power, i.e., show that:

a) Every PDA accepting by empty stack can be simulated by an equivalent PDA accepting by final state.

b) Every PDA accepting by final state can be simulated by an equivalent PDA accepting by empty stack.

Exercise 9: Consider the following language:

$$L = \{ ww \mid w \in \{a, b\}^* \}$$

a) Show that L is not context-free.
b) Show that the complement of L is context-free.

Exercise 10: For each of the following languages determine to which of the following classes it belongs: (a) regular, (b) context-free but not regular, (c) item non-context-free.

- $L_1 = \{ w \in \{a, b\}^* \mid |w|_a = |w|_b \}$
- $L_2 = \{ w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0 \}$
- $L_3 = \{ w \in \{a, b\}^* \mid w \text{ contains subword } abba \}$
- $L_4 = \{ w \in \{a, b, c\}^* \mid |w|_a = |w|_b = |w|_c \}$
- $L_5 = \{ w \in \{a, b\}^* \mid |w|_a \text{ is a prime} \}$
- $L_6 = \{ 0^m 1^n \mid m \leq 2n \}$
- $L_7 = \{ 0^m 1^n 0^m \mid m = 2n \}$