Tutorial 5

Exercise 1: Consider the following context-free grammar:

$$\begin{array}{rcl} S & \longrightarrow & aBb \mid AB \\ A & \longrightarrow & bAb \mid a \\ B & \longrightarrow & \varepsilon \mid aABb \end{array}$$

a) Give (some) derivation of word $babaab$ in this grammar.

b) Draw the corrensponding derivation tree.

c) Write the left and right derivations corresponding to the derivation tree drawn in the previous point.

Exercise 2: Construct context-free grammars for all following languages:

- $L_1 = \{w \in \{a, b, c\}^* \mid w$ contains subword $babb\}$
- $L_2 = \{0^n 1^m \mid 1 \leq n < m\}$
- $L_3 = \{a^n b^m a^{n+2} \mid m, n \in \mathbb{N}\}$
- $L_4 = \{w \in \{0, 1\}^* \mid w = w^R\}$
- $L_5 = \{w \in \{0, 1\}^* \mid |w|_0 > 1,\ |w|_1 \leq 2\}$
- $L_6 = \{0^n w w^R 1^n \mid w \in \{0, 1\}^*,\ n \in \mathbb{N}\}$
- $L_7 = \{w \in \{a, b\}^* \mid$ in w, every a is directly followed by b, or $w = b^n a^m$, where $0 \leq m \leq n\}$
- $L_8 = \{u v^R v \mid u, v \in \{0, 1\}^*,\ |u|_0 \bmod 4 = 2,\ u$ ends with suffix 101 and v contains subword $10\}$
- $L_9 = \{w \in \{a, b, c\}^* \mid$ every sequence of a's is directly followed by a sequence of b's, which is twice as long$\}$

Exercise 3: Decide for the following pairs of grammars if both grammars generate the same language. Justify your answers.

a) $S \longrightarrow aaSbb \mid ab \mid aabb$ $\qquad S \longrightarrow aSb \mid ab$

b) $S \longrightarrow aaSbb \mid ab \mid \varepsilon$ $\qquad S \longrightarrow aSb \mid ab$

c) $S \longrightarrow aaSb \mid ab \mid \varepsilon$ $\qquad S \longrightarrow aSb \mid aab \mid \varepsilon$

Exercise 4: Construct a context-free grammar for the language L over the alphabet $\Sigma = \{(,), [,]\}$ consisting of all "correctly parenthesized" expressions. As correctly parenthesized expressions we consider those sequences of symbols where each left parenthesis has a corresponding right parenthesis of the same type, and where parenthesis do not "cross" (i.e., coresponding pairs of parenthesis are composed correctly).

Exercise 5: Construct a context free grammar G such that $L(G) = L_1 \cdot L_2$ where

- $L_1 = \{ w \in \{a,b\}^* \mid w \text{ contains subword } bab \}$
- $L_2 = \{ a^n u \mid u \in \{a,b\}^* \text{ and } 1 \leq n \leq |u| \leq 2n \}$

Exercise 6: Propose a syntax for writing simple arithmetic expressions as words over the alphabet

$$\Sigma = \{\mathtt{A}, \mathtt{B}, \ldots, \mathtt{Z}, \mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}, \mathtt{0}, \mathtt{1}, \ldots, \mathtt{9}, \mathtt{.}, \mathtt{+}, \mathtt{-}, \mathtt{*}, \mathtt{/}, \mathtt{(}, \mathtt{)}\}.$$

and describe the proposed syntax by a context-free grammar.

Exercise 7: Try to describe as precisely as possible a characteristic property of words of the language generated by the following context-free grammar:

$$S \longrightarrow a \mid bSS$$

Exercise 8: Find out whether it holds for the following context-free grammar G that $L(G) \neq \emptyset$, i.e., whether it is possible to generate at least one word from nonterminal S using grammar G:

$$\begin{aligned} S &\longrightarrow aS \mid AB \mid CD \\ A &\longrightarrow aDb \mid AD \mid BC \\ B &\longrightarrow bSb \mid BB \\ C &\longrightarrow BA \mid ASb \\ D &\longrightarrow ABCD \mid \varepsilon \end{aligned}$$

Exercise 9: Consider language

$$L = \{ w \in \{a,b\}^* \mid |w| \geq 1, |w|_a = |w|_b \}.$$

i.e., the language containing nonempty words with the same number of a's as b's.

At first, try to characterize words from $L^2 = L \cdot L$. Is it true that $L^2 = L$? Does at least one of the inclusions $L \subseteq L^2$ and $L^2 \subseteq L$ hold?

Try to characterize the words from $L - L^2$.

Using the previous ideas, construct a context-free grammar for the language L.

***Exercise 10:** Construct a context-free grammar generating the language of all palindroms over the alphabet $\{a, b\}$ whose length is a multiple of four, resp. of three, i.e., for the languages

- $L_1 = \{ w \in \{a,b\}^* \mid w = w^R \text{ and } |w| \bmod 4 = 0 \}$,
- $L_2 = \{ w \in \{a,b\}^* \mid w = w^R \text{ and } |w| \bmod 3 = 0 \}$.