Tutorial 2

Exercise 1: Consider the following languages:

$L_1 = \{w \in \{0,1\}^* \mid \text{in } w \text{ is every } 0 \text{ (directly) followed by } 1\}$
$L_2 = \{w \in \{0,1\}^* \mid w = w^R\}$

a) Enumerate the first 5 words of each of languages L_1, L_2 (the smallest words with respect to order $<_L$).

b) Enumerate the first 5 words of each of languages $\overline{L_1}$, $\overline{L_2}$.

c) Enumerate the first 5 words of language $L_1 \cap L_2$.

d) Enumerate the first 5 words of language $L_1 \cup L_2$.

Exercise 2: Consider languages over $\{a, b\}$. Write down all the words in the concatenation of $L_1 = \{\varepsilon, abb, bba\}$ and $L_2 = \{a, b, abba\}$.

Exercise 3: Consider languages over the alphabet $\{0,1\}$. Write down all words in the concatenation

$$\{0, 001, 111\} \cdot \{\varepsilon, 01, 0101\}$$

Exercise 4: Consider languages over the alphabet $\{0,1\}$. Describe the language of all words in the iteration $\{00, 111\}^*$ and write the first 10 words of the language.

Exercise 5: Consider the following languages:

$L_1 = \{w \in \{0,1\}^* \mid |w|_1 \leq 1\}$
$L_2 = \{w \in \{0,1\}^* \mid w = w^R\}$

Describe the words in the language $L_1 \cap L_2$.

Exercise 6: Decide which of the following relations are valid for all languages L_1, L_2, L_3:

a) $(L_1 \cup L_2) \cdot L_3 = (L_1 \cdot L_3) \cup (L_2 \cdot L_3)$?

b) $(L_1 \cap L_2) \cdot L_3 = (L_1 \cdot L_3) \cap (L_2 \cdot L_3)$?

c) $(L_1 \cap L_2)^* = L_1^* \cap L_2^*$?

d) $(L_1 \cup L_2)^* = L_1^* \cdot (L_2 \cdot L_1^*)^*$?

*__Exercise 7:__ Prove that for each language L is $L \cdot L \subseteq L$ iff $L^* = L \cup \{\varepsilon\}$.

__Exercise 8:__ Construct DFA A_1, A_2 such that:

$$L(A_1) = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0\}$$
$$L(A_2) = \{w \in \{a, b\}^* \mid \text{every occurence of } b \text{ in } w \text{ is followed with } a\}$$

Using automata A_1, A_2, construct DFA accepting the following languages:

a) $L_1 = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0$ and every occurence of b in w is followed with $a\}$

b) $L_2 = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0$ or every occurrence of b in w is followed with $a\}$

c) $L_3 = \{w \in \{a, b\}^* \mid$ some occurrence of b in w is not followed with $a\}$

d) $L_4 = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0$ and some occurrence of b in w is not followed with $a\}$

e) $L_5 = \{w \in \{a, b\}^* \mid$ if $|w|_a \bmod 2 = 0$ then every occurrence of b in w is followed with $a\}$

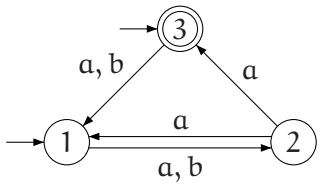
f) $L_6 = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0$ iff every occurrence of b in w is followed with $a\}$

__Exercise 9:__ Construct NFA accepting the following languages:

a) $L_1 = \{w \in \{a, b, c\}^* \mid |w|_a = 0 \vee |w|_b \bmod 2 = 0 \vee |w|_c \bmod 3 = 2\}$
b) $L_2 = \{w \in \{a, b, c\}^* \mid |w| \geq 8$ and the eighth symbol from the end of word w is $a\}$
c) $L_3 = \{abaabw \mid w \in \{a, b\}^*\}$
d) $L_4 = \{wabaab \mid w \in \{a, b\}^*\}$
e) $L_5 = \{w_1 abaabw_2 \mid w_1, w_2 \in \{a, b\}^*\}$

__Exercise 10:__ Construct DFA equivalent to the given NFA:

a)



b)