

Přehled pravidel

Základní pravidla pro logické spojky:

$$\begin{array}{l}
 \text{Assm: } \frac{}{\Gamma, A, \Delta \vdash A} \qquad \text{Ant: } \frac{\Gamma \vdash A}{\Gamma' \vdash A} (\Gamma \subseteq \Gamma') \\
 \wedge\text{i: } \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \qquad \wedge\text{e}_1: \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \qquad \wedge\text{e}_2: \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \\
 \vee\text{i}_1: \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \qquad \vee\text{i}_2: \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \qquad \vee\text{e: } \frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \\
 \rightarrow\text{i: } \frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \qquad \rightarrow\text{e: } \frac{\Gamma \vdash A \rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B} \\
 \leftrightarrow\text{i: } \frac{\Gamma, A \vdash B \quad \Gamma, B \vdash A}{\Gamma \vdash A \leftrightarrow B} \qquad \leftrightarrow\text{e}_1: \frac{\Gamma \vdash A \leftrightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B} \\
 \qquad \qquad \qquad \leftrightarrow\text{e}_2: \frac{\Gamma \vdash A \leftrightarrow B \quad \Gamma \vdash B}{\Gamma \vdash A} \\
 \neg\text{i: } \frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \qquad \neg\text{e: } \frac{\Gamma \vdash \neg A \quad \Gamma \vdash A}{\Gamma \vdash \perp} \\
 \top\text{i: } \frac{}{\vdash \top} \qquad \perp\text{e: } \frac{\Gamma \vdash \perp}{\Gamma \vdash A} \\
 \neg\neg\text{e: } \frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}
 \end{array}$$

Některá užitečná odvozená pravidla:

$$\begin{array}{l}
 \text{Ctr: } \frac{\Gamma, A \vdash B \quad \Gamma, A \vdash \neg B}{\Gamma \vdash \neg A} \qquad \text{CtrN: } \frac{\Gamma, \neg A \vdash B \quad \Gamma, \neg A \vdash \neg B}{\Gamma \vdash A} \\
 \text{CtrA: } \frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash B} \qquad \neg\neg\text{i: } \frac{\Gamma \vdash A}{\Gamma \vdash \neg\neg A} \\
 \text{Cp(a): } \frac{\Gamma, A \vdash B}{\Gamma, \neg B \vdash \neg A} \qquad \text{Cp(b): } \frac{\Gamma, A \vdash \neg B}{\Gamma, B \vdash \neg A} \\
 \text{Cp(c): } \frac{\Gamma, \neg A \vdash B}{\Gamma, \neg B \vdash A} \qquad \text{Cp(d): } \frac{\Gamma, \neg A \vdash \neg B}{\Gamma, B \vdash A} \\
 \text{Ch: } \frac{\Gamma \vdash A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \qquad \text{PC: } \frac{\Gamma, A \vdash B \quad \Gamma, \neg A \vdash B}{\Gamma \vdash B}
 \end{array}$$

Pravidla pro kvantifikátory a pro rovnost:

$$\forall e: \frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[t/x]}$$

$$\forall i: \frac{\Gamma \vdash A[y/x]}{\Gamma \vdash \forall x A} (y \notin \text{free}(\Gamma, \forall x A))$$

$$\exists i: \frac{\Gamma \vdash A[t/x]}{\Gamma \vdash \exists x A}$$

$$\exists e: \frac{\Gamma \vdash \exists x A \quad \Gamma, A[y/x] \vdash B}{\Gamma \vdash B} (y \notin \text{free}(\Gamma, \exists x A, B))$$

$$=i: \frac{}{\vdash t = t}$$

$$=e: \frac{\Gamma \vdash t = t' \quad \Gamma \vdash A[t/x]}{\Gamma \vdash A[t'/x]}$$