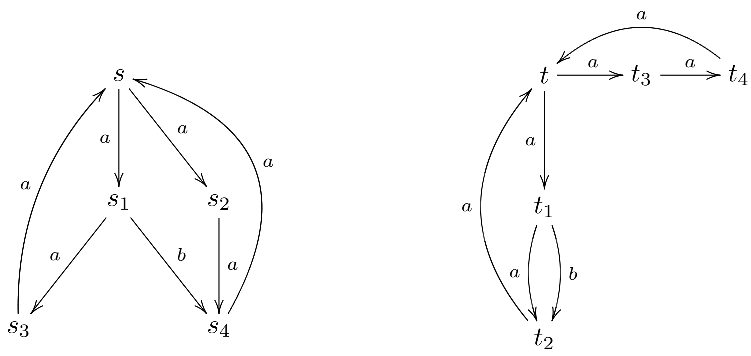
Tutorial 3

Exercise 1*

Consider the following labelled transition system.



Show that $s \sim t$ by finding a strong bisimulation R containing the pair (s, t).

Exercise 2*

Consider the CCS processes P and Q defined by:

$$\begin{aligned} P &\stackrel{\text{def}}{=} a.P_1 \\ P_1 &\stackrel{\text{def}}{=} b.P + c.P \end{aligned}$$

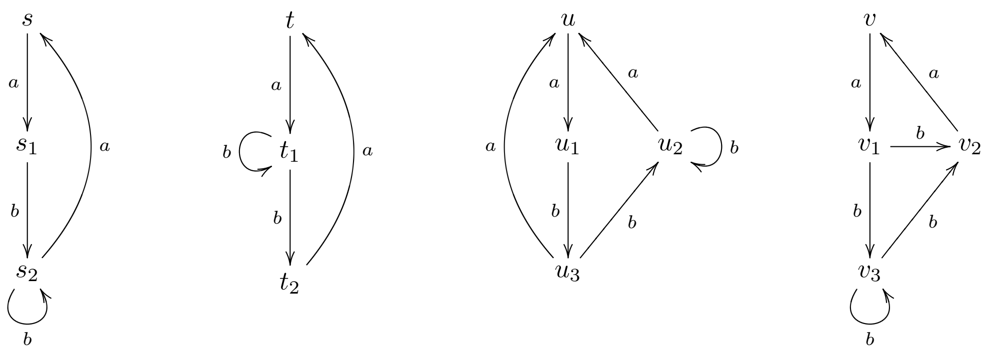
and

$$\begin{aligned} Q &\stackrel{\text{def}}{=} a.Q_1 \\ Q_1 &\stackrel{\text{def}}{=} b.Q_2 + c.Q \\ Q_2 &\stackrel{\text{def}}{=} a.Q_3 \\ Q_3 &\stackrel{\text{def}}{=} b.Q + c.Q_2 \quad . \end{aligned}$$

Show that $P \sim Q$ holds by finding an appropriate strong bisimulation.

Exercise 3*

Consider the following labelled transition system.



Decide whether $s \stackrel{?}{\sim} t$, $s \stackrel{?}{\sim} u$, and $s \stackrel{?}{\sim} v$. Support your claims by giving a universal winning strategy either for the attacker (in the negative case) or the defender (in the positive case). In the positive case you can also define a strong bisimulation relating the pair in question.

Exercise 4

Prove that for any CCS processes P and Q the following laws hold:

- $P \,|\, Nil \sim P$
- $P + Nil \sim P$
- $P \,|\, Q \sim Q \,|\, P$

Hint: define appropriate binary relations on processes and prove that they are strong bisimulations.

Exercise 5

Argue that any two strongly bisimilar processes have the same sets of traces, i.e., that

$$s \sim t \text{ implies } Traces(s) = Traces(t).$$

Hint: you can find useful the game characterization of strong bisimilarity.

Exercise 6 (optional)

Is it true that any relation of strong bisimilarity must be reflexive, transitive and symmetric? If yes then prove it, if not then give counter examples, i.e.

- define an LTS and a binary relation on states which is not reflexive but it is a strong bisimulation
- define an LTS and a binary relation on states which is not symmetric but it is a strong bisimulation
- define an LTS and a binary relation on states which is not transitive but it is a strong bisimulation.

Exercise 7 (optional)

Argue that $s \sim t$ iff the defender has a winning strategy in the strong bisimulation game starting from the pair (s, t).

Hint: show that from knowing defender's universal winning strategy you can find a strong bisimulation and that given a strong bisimulation you can define defender's universal winning strategy.