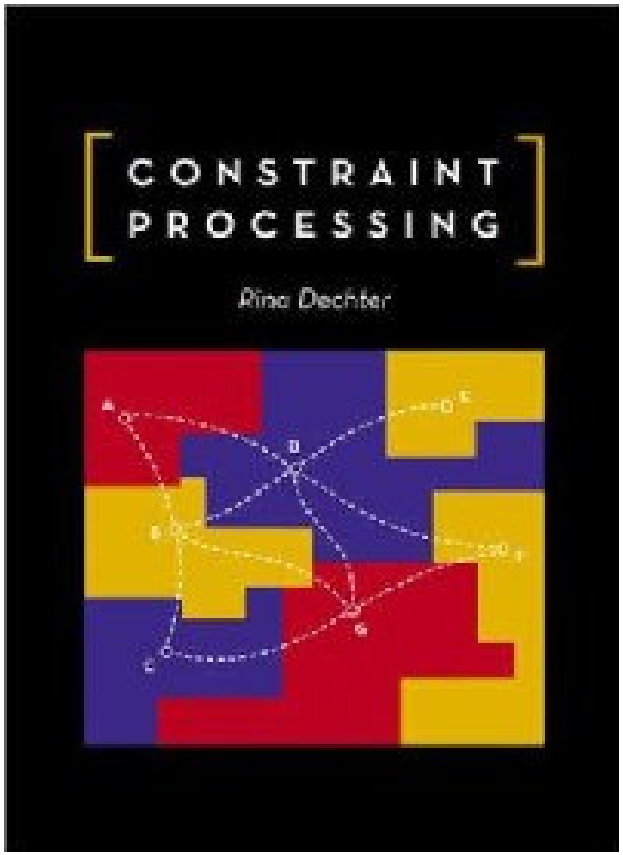
Constraint Processing

Rina Dechter,

Constraint Processing,

Morgan Kaufmann

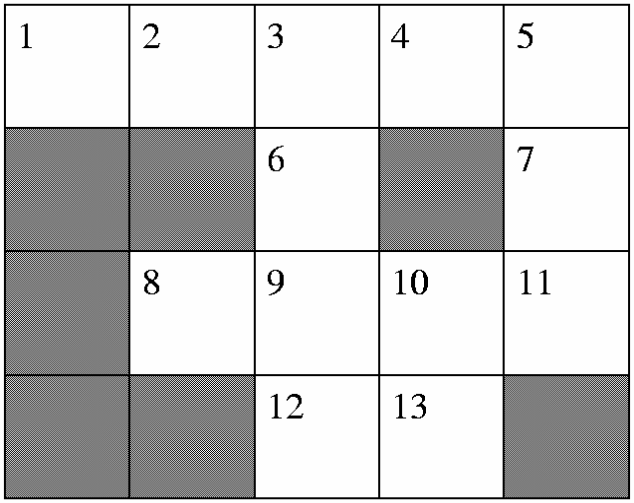


Constraint network

- A constraint network is defined by:
  - A set of variables
  - A domain of values for each variable
  - A set of constraints

Crossword puzzle

- Variables: x_1, …, x_{13}
- Domains: letters
- Constraints: words from

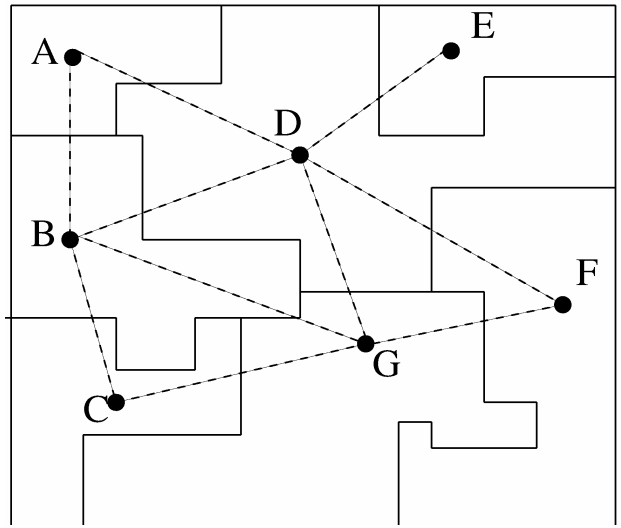


{HOSES, LASER, SHEET, SNAIL, STEER, ALSO, EARN, HIKE, IRON, SAME, EAT, LET, RUN, SUN, TEN, YES, BE, IT, NO, US}

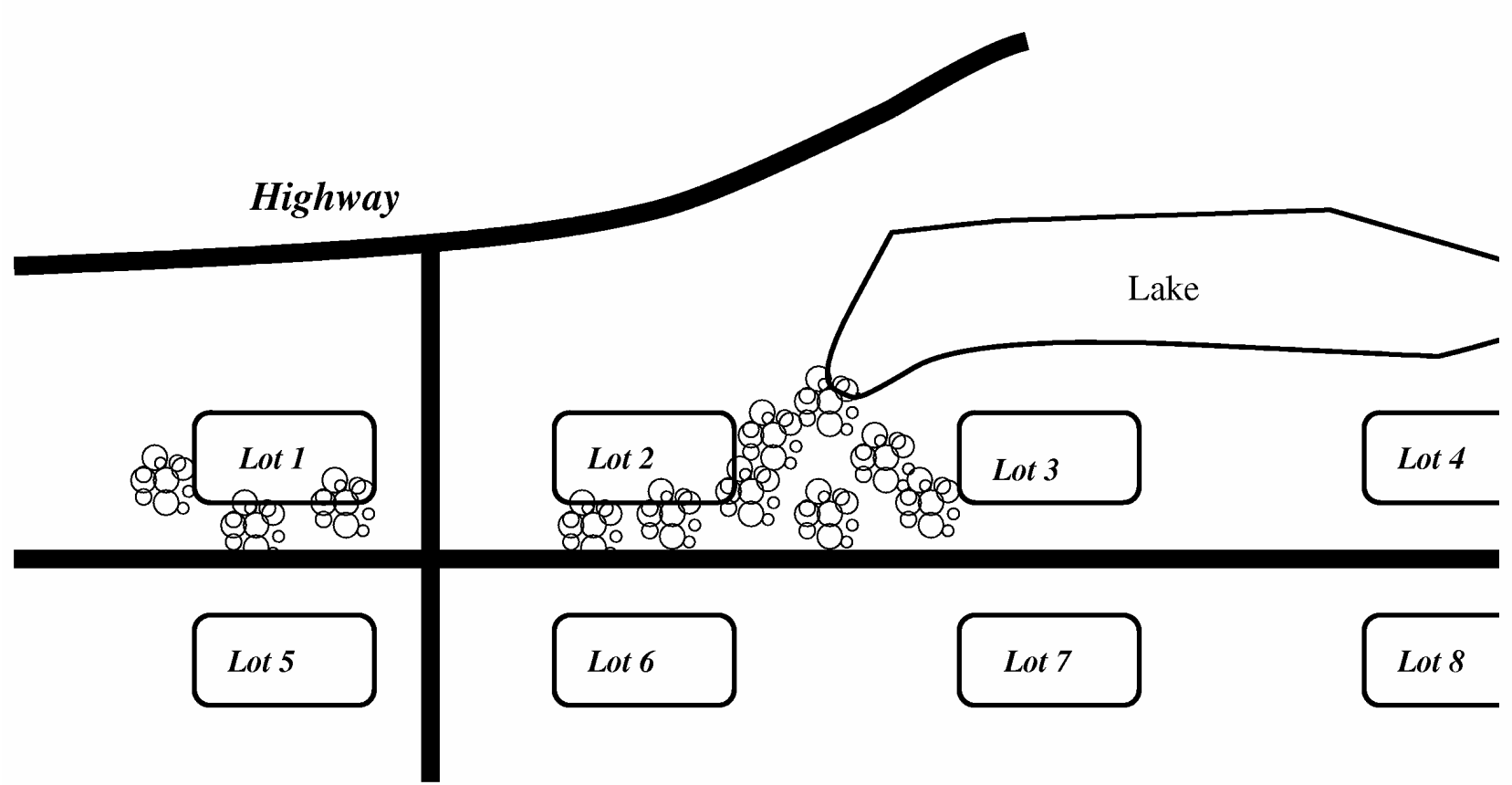
Map coloring; k-colorability

- Variables: A, B, …, G
- Domains: k colors
- Constraints:

  Adjacent nodes should be different

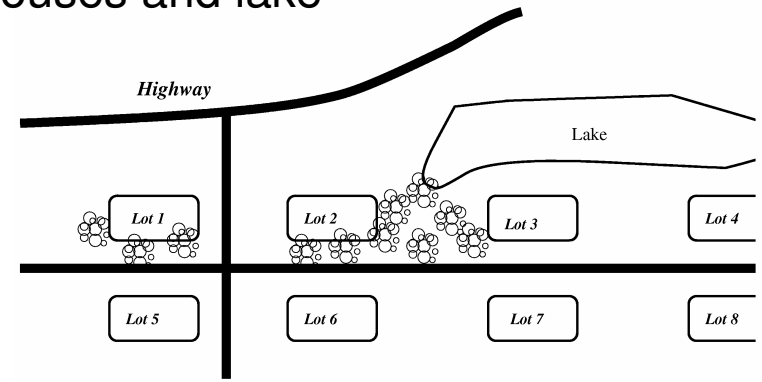


Configuration and design



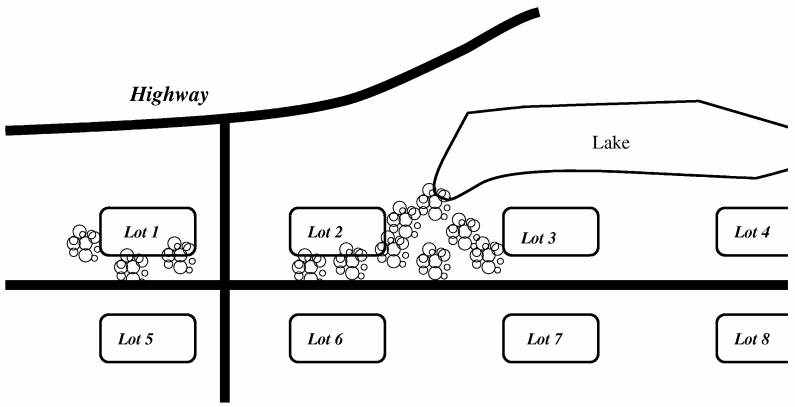
Configuration and design

- Want to build: recreation area, apartments, houses, cemetery, dump
  - Recreation area near lake
  - Steep slopes avoided except for recreation area
  - Poor soil avoided for developments
  - Highway far from apartments, houses and recreation
  - Dump not visible from apartments, houses and lake
  - Lots 3 and 4 have poor soil
  - Lots 3, 4, 7, 8 are on steep slopes
  - Lots 2, 3, 4 are near lake
  - Lots 1, 2 are near highway



Configuration and design

- Variables: *Recreation, Apartments, Houses, Cemetery, Dump*
- Domains: {1, 2, …, 8}
- Constraints: derived from conditions



Huffman-Clowes junction labelings (1975)

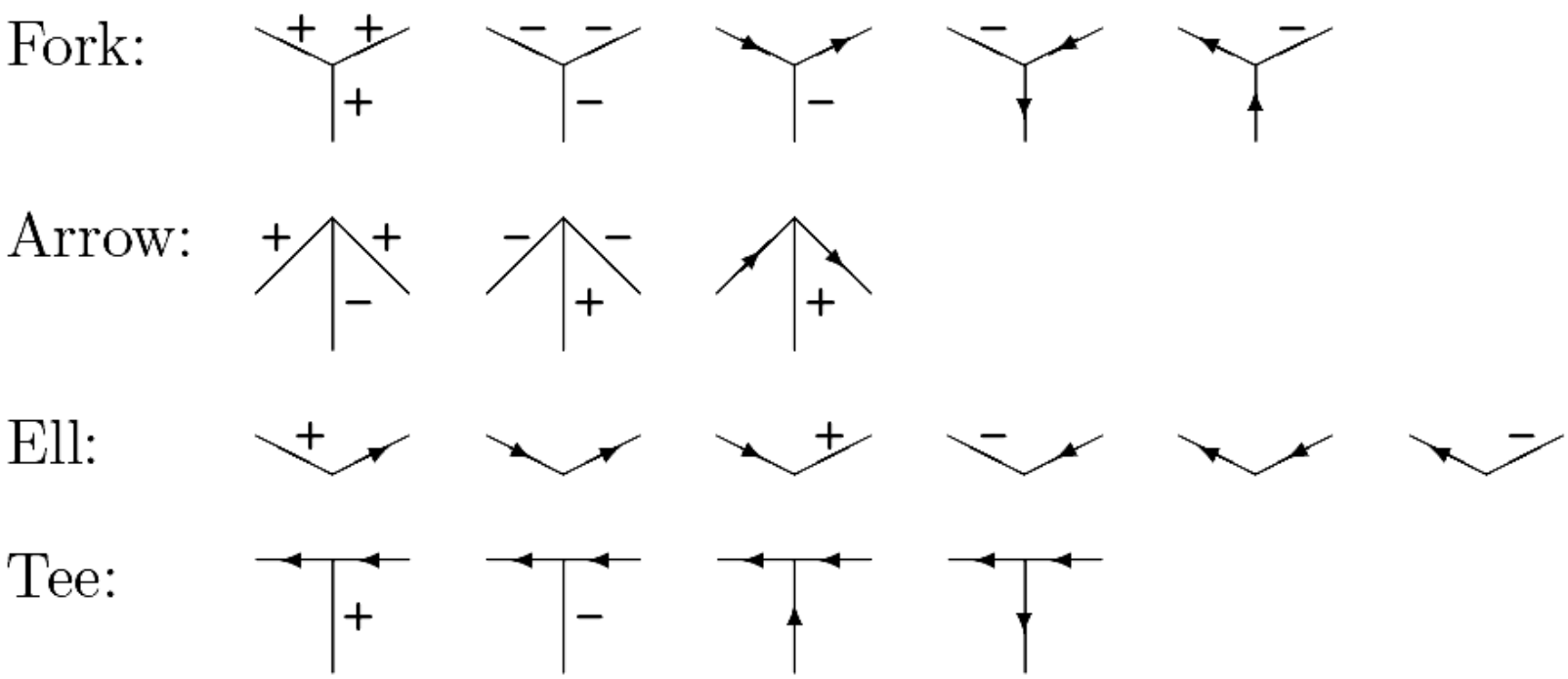
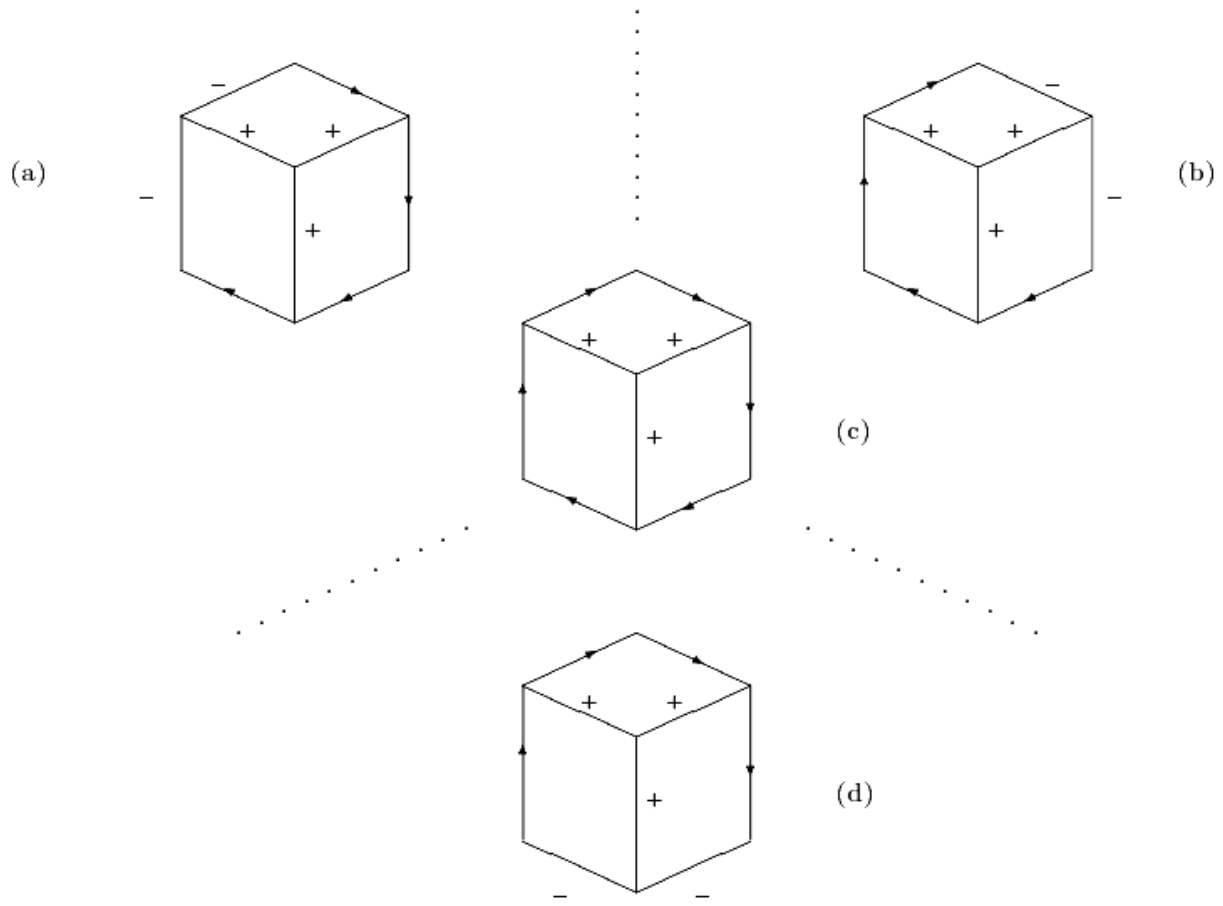


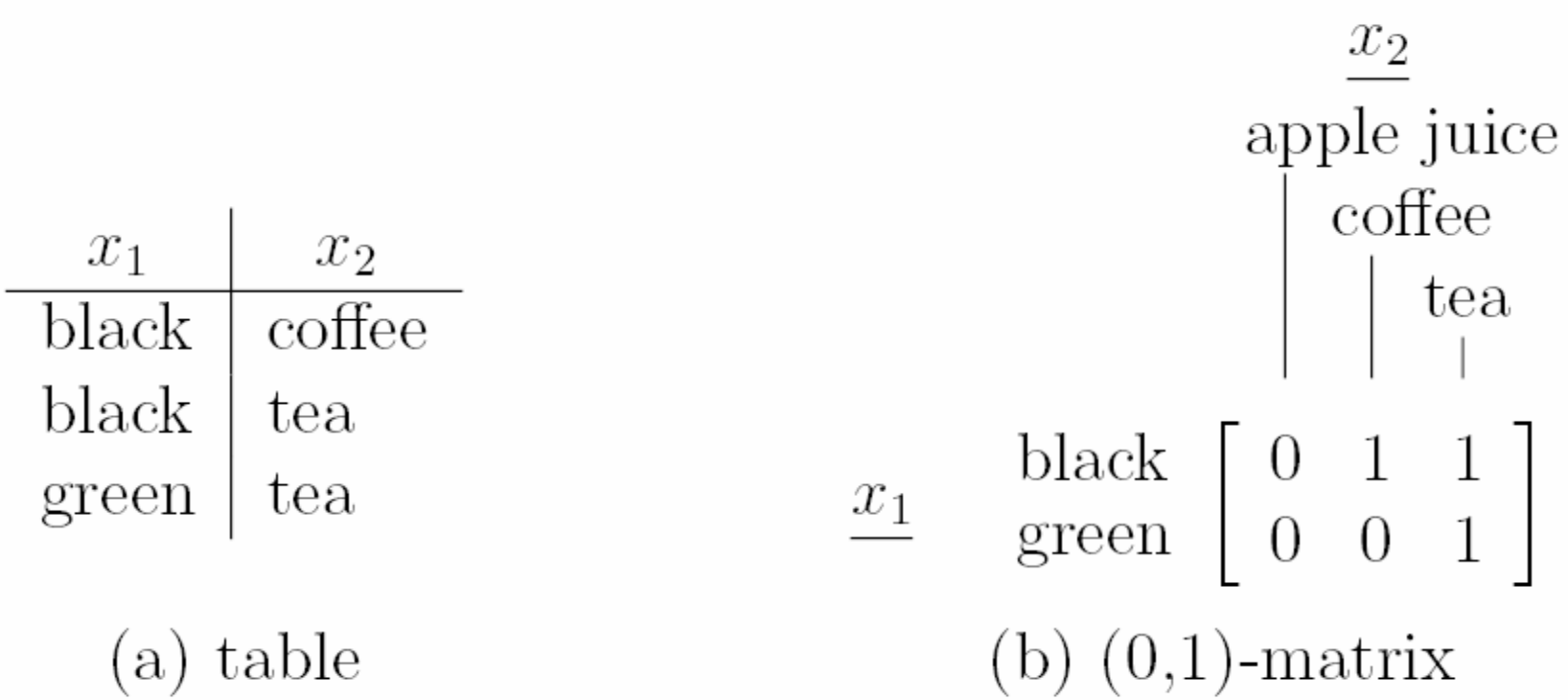
Figure 1.5: Solutions: (a) stuck on left wall, (b) stuck on right wall, (c) suspended in mid-air, (d) resting on floor.



Mathematical background

- Sets, domains, tuples
- Relations
- Operations on relations
- Graphs
- Complexity

Figure 1.6: Two graphical views of relation *R* = {(black, coffee), (black, tea), (green, tea)}.



Operations with relations

- Intersection
- Union
- Difference
- Selection
- Projection
- Join
- Composition

Figure 1.7: Three relations.

x_1	x_2	x_3
a	b	c
b	b	c
c	b	c
c	b	s

(a) Relation R

x_1	x_2	x_3
b	b	c
c	b	c
c	n	n

(b) Relation R'

x_2	x_3	x_4
a	a	1
b	c	2
b	c	3

(c) Relation R''

Figure 1.8: Example of set operations intersection, union, and difference applied to relations.

x_1	x_2	x_3
a	b	c
b	b	c
c	b	c
c	b	s

(a) Relation R

x_1	x_2	x_3
b	b	c
c	b	c
c	n	n

(b) Relation R'

x_2	x_3	x_4
a	a	1
b	c	2
b	c	3

(c) Relation R''

x_1	x_2	x_3
b	b	c
c	b	c

(a) $R \cap R'$

x_1	x_2	x_3
a	b	c
b	b	c
c	b	c
c	b	s
c	n	n

(b) $R \cup R'$

x_1	x_2	x_3
a	b	c
c	b	s

(b) $R - R'$

Figure 1.9: Example of selection, projection, and join operations on relations.

x_1	x_2	x_3
a	b	c
b	b	c
c	b	c
c	b	s

(a) Relation R

x_1	x_2	x_3
b	b	c
c	b	c
c	n	n

(b) Relation R'

x_2	x_3	x_4
a	a	1
b	c	2
b	c	3

(c) Relation R''

x_1	x_2	x_3
b	b	c
c	b	c

(a) $\sigma_{x_3=c}(R')$

x_2	x_3
b	c
n	n

(b) $\pi_{\{x_2,x_3\}}(R')$

x_1	x_2	x_3	x_4
b	b	c	2
b	b	c	3
c	b	c	2
c	b	c	3

(c) $R' \bowtie R''$

Figure 1.10: Two graphs: (a) undirected and (b) directed.