Normed BPA vs. normed BPP revisited

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22 August 2008
CONCUR'08

Brief history . . .

- After the models like FA, PDA, CFG were defined, decidability and complexity questions regarding language equivalence have been studied . . .
- For example:
  - NFA – PSPACE-complete
  - CFG, PDA – undecidable

CFG grammar

$$V = \{A, B\}$$
$$\mathcal{A} = \{a, b\}$$

$$A \longrightarrow b$$
$$A \longrightarrow bAB$$
$$B \longrightarrow a$$

BPA system (sequential composition)

$$V = \{A, B\}$$
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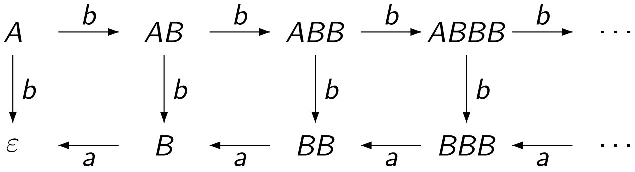
$$A \longrightarrow b$$
$$A \longrightarrow bAB$$
$$B \longrightarrow a$$

$$A \xrightarrow{b} \varepsilon$$
$$A \xrightarrow{b} AB$$
$$B \xrightarrow{a} \varepsilon$$

BPA system (sequential composition)

$$V = \{A, B\} \qquad A \longrightarrow b \qquad A \xrightarrow{b} \varepsilon$$
$$\mathcal{A} = \{a, b\} \qquad A \longrightarrow bAB \qquad A \xrightarrow{b} AB$$
$$B \longrightarrow a \qquad B \xrightarrow{a} \varepsilon$$

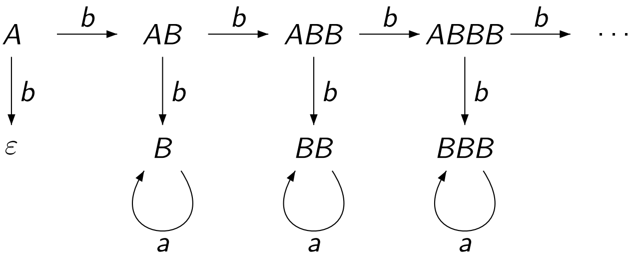
- Using left-most derivation it defines a LTS:



- $L(\alpha) = \{w \in \mathcal{A}^* \mid \alpha \longrightarrow^* w\} = \{w \in \mathcal{A}^* \mid \alpha \xrightarrow{w} \varepsilon\}$
- System is normed if $(\forall A \in V)(\exists w \in \mathcal{A}^*) : A \xrightarrow{w} \epsilon$

Unnormed BPA system

$$V = \{A, B\} \qquad A \xrightarrow{b} \varepsilon$$
$$\mathcal{A} = \{a, b\} \qquad A \xrightarrow{b} AB$$
$$B \xrightarrow{a} B$$



Brief history . . . (continuation)

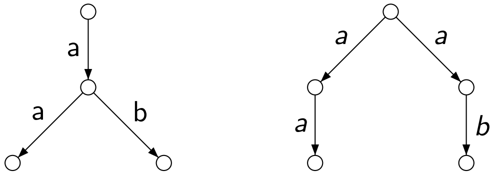
- In 1980s, bisimilarity . . . fundamental behavioral equivalence

Definition (Bisimulation)

Given an LTS $(S, \mathcal{A}, \longrightarrow)$, a binary relation $\mathcal{R} \subseteq S \times S$ is a **bisimulation** iff for each $(s, t) \in \mathcal{R}$ and $a \in \mathcal{A}$ we have:

- $\forall s' \in S : s \xrightarrow{a} s' \Rightarrow (\exists t' : t \xrightarrow{a} t' \wedge (s', t') \in \mathcal{R})$, and
- $\forall t' \in S : t \xrightarrow{a} t' \Rightarrow (\exists s' : s \xrightarrow{a} s' \wedge (s', t') \in \mathcal{R})$.

States s and t are **bisimulation equivalent (bisimilar)**, written $s \sim t$, iff they are related by some bisimulation.



Brief history . . . (continuation)

- Also for bisimilarity, decidability and complexity questions are a natural topic to study
- . . .
  - NFA – polynomial
  - normed BPA – decidable [Baeten, Bergstra, Klop, JACM 1993]
- This was a seminal paper for a line of research . . .

Example of a BPP system

Parallel composition is natural alternative to sequential

$$V = \{A, B\} \qquad A \xrightarrow{b} \varepsilon$$
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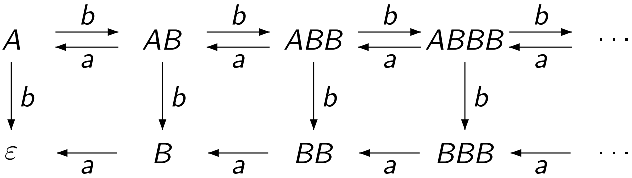

As parallel composition is commutative and associative, Parikh images of sequence can be considered as states of LTS

Basic Parallel Processes (BPP)

$$V = \{A, B\} \qquad A \xrightarrow{b} \varepsilon$$
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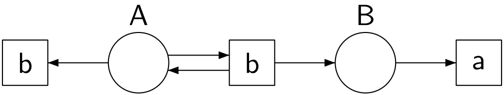

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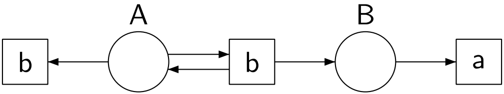

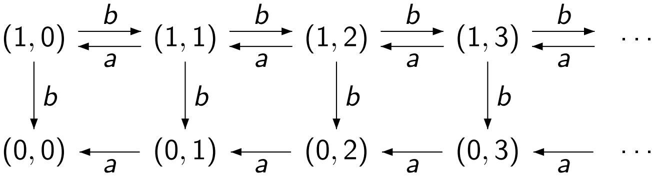

Known results

For BPA:

- Bisimilarity on BPA is in 2-EXPTIME and PSPACE-hard
- Bisimilarity on normed BPA is in $O(n^8 polylog\ n)$

For BPP:

- Bisimilarity on BPP is PSPACE-complete
- Bisimilarity on normed BPP is in $O(n^3)$

PA

- Both sequential and parallel composition are allowed
- Decidability of bisimilarity is open question
  - (adding communication - Turing powerfull)
- Normed PA
  - Decidable [Hirshfeld, Jerrum, 1999]
  - Quite complicated proof
  - Most important part - characterising when $P_1 \cdot P_2 \sim Q_1 || Q_2$

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- Bisimilarity between BPA process and BPP process is a simple subcase
- For normed BPA and BPP - decidable (in exponential time) [Černá, Křetínský and Kučera]
- For general BPA and BPP - decidable [Jančar, Kučera, Moller]

Main problem

Problem nBPA-nBPP-bisim

Instance: A BPP process definition Δ with initial marking M_0 and a BPA process definition Σ with initial configuration α_0

Question: Is $M_0 \sim \alpha_0$?

Main result

Problem nBPA-nBPP-bisim is decidable in polynomial time.

Algorithm - sketch

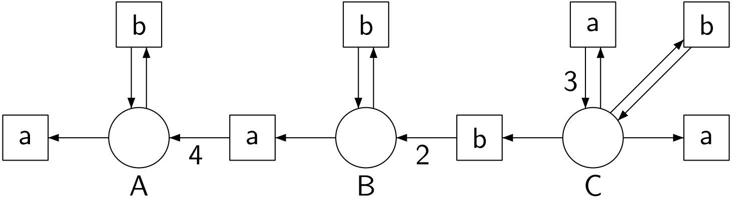
- Transform (M_0, Δ) to bisimilar (M_0', Δ') into a special form (called prime form)
- Check certain conditions characterising when there exists a BPA process bisimilar with M_0 (which possibly leads to an answer $\alpha_0 \not\sim M_0$)
- Construct BPA Σ' with initial configuration α_0' such that $\alpha_0' \sim M_0'$, if the number of variables exceeds "some bound" end with answer $\alpha_0 \not\sim M_0$
- Check whether $\alpha_0 \sim \alpha_0'$

Prime form of BPP

- Every BPP can be transformed into a special form where bisimilarity coincides with identity

$$M \sim M' \text{ iff } M = M'$$

Example of BPP which is not in a prime form:



$(5, 0, 0) \sim (0, 1, 0)$

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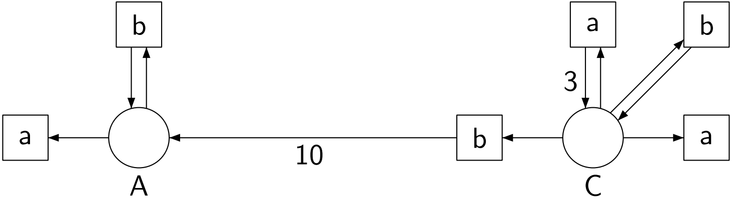


Transformation of BPP into the prime form

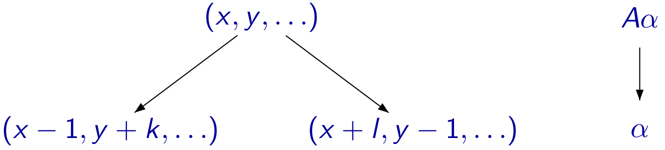
- It is possible to use algorithm implicitly present in [Hirshfeld, Jerrum, Moller, 1996]
    - it is polynomial
    - precise complexity has not been analyzed
- We suggest an alternative algorithm
    - it is based on dd-functions
    - the transformation is done in time $O(n^3)$
    - we do not go into details in this presentation

A combination of observations

- The prime form allowed us to achieve our result by a combination of simple observations
- Those observations lead to conditions on BPP potentially excluding the existence of a bisimilar BPA

An example of an observation

If $A\alpha \sim M$ and M marks at least two places then $\|A\| \geq 2$.
Proof by contradiction: $A\alpha \sim M$, M marks at least two places, $\|A\| = 1$



Other observations

- If $\alpha \sim M$ and M marks at least two places then number of tokens in M is at most $|V|$.
- If $\alpha \sim M$ then $M(p) \leq |V|$ for every non-SF-place p.

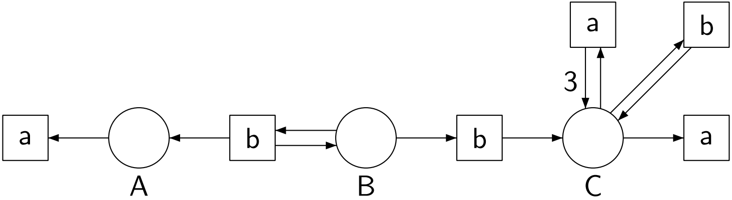
Single final place

Definition

A place p is called a **single final place** (SF-place) if no transition which takes a token from p gives a token to some other place.

Remark

$\|p\| = 1$ for every SF-place p



A is SF-place, C is growing SF-place and B is non-SF-place

Conditions on BPP excluding a bisimilar BPA

If one of the following conditions hold for (M_0, Δ) there is not any (α_0, Σ) such that $\alpha_0 \sim M_0$:

1. A non-SF-place is unbounded
2. $M_0 \longrightarrow^* M$ such that M has at least two marked places and $M(p) \geq 1$ for some growing SF-place p
3. A non-growing SF-place p is unbounded

If no of those conditions holds, there are only two types of reachable markings:

- Tokens are only in bounded places
- All tokens are in one SF-place

BPA construction

- Construct reachability graph of M_0 - markings with all tokens in one SF-place are "frozen"
- Construct BPA Σ' where:
  - a variable A_M for each unfrozen marking
  - a variable I_p for each SF-place p
  - rules $A_M \xrightarrow{a} A_{M'}, A_M \xrightarrow{a} (I_p)^k, I_p \xrightarrow{a} (I_p)^k$
- Constructed BPA can possibly be of exponential size

BPA construction

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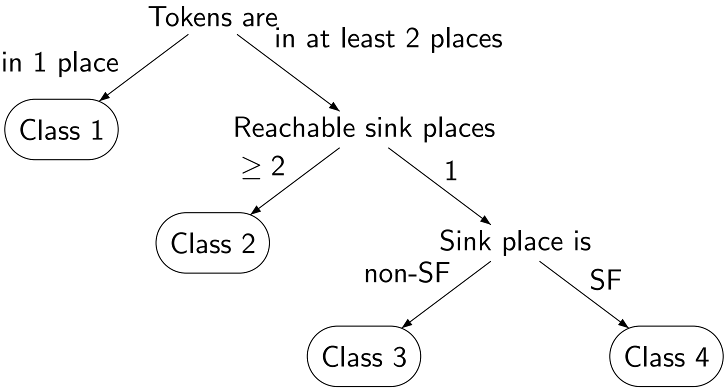
- Constructed BPA can possibly be of exponential size

- Our goal is to check bisimilarity with the given Σ, we can use it for a bound
- If a number of unfrozen markings exceeds $4N^2$ where N is maximum of $\{|V_\Sigma|, |P_{\Delta'}|\}$ end with answer $\alpha_0 \not\sim M_0$

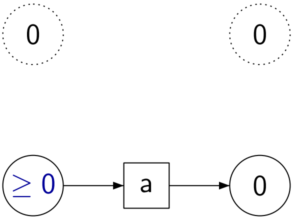
Bound on the number of "unfrozen" markings

- Divide "unfrozen" markings into 4 classes
- Show that the size of each class is bounded by N^2



Class 1.

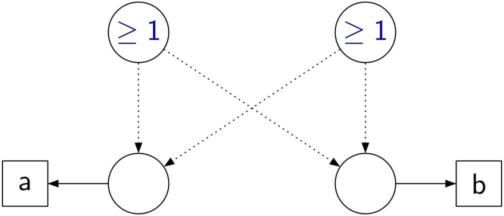
Markings with all tokens in one (non-SF) place



- Number of tokens is bounded by $|V_\Sigma|$
- Number of places is $|P_\Delta|$
- There is at most $|V_\Sigma| \cdot |P_\Delta|$ markings in class 1.

Class 2.

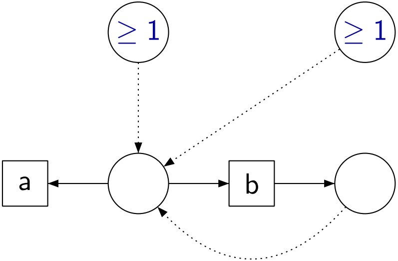
Markings with at least two marked places, at least two sink places with norm 1 are reachable



- If $\alpha \sim M$ for M form class 2 then $\alpha = A$
- Number of markings is at most $|V| \leq N^2$

Class 3.

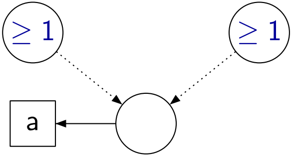
Markings with at least two marked places, only one sink places with norm 1 are reachable, the sink place is a non-SF-place



- If $A\alpha \sim M$ for M form class 3 then $\|\alpha\| \leq 1$
- The number of markings in Class 3 is at most $|V|^2 \leq N^2$.

Class 4.

Markings with at least two marked places, only one sink places with norm 1 are reachable, the sink place is a SF-place



- Let $A\alpha \sim M$ for M from Class 4, p is SF-place.
- $\alpha \sim I^k$ where $k = \|\alpha\|$ and $I \in V$, $I \sim p$
- There is M' reachable from M by norm reducing steps, M' does not have all tokens in p, every norm reducing transition from M' leads to marking with all tokens in p
- It follows, that M' has only 1 token ($|P|$ possibilities for M')
- The number of markings in Class 4 is at most $|V| \cdot |P| \leq N^2$.

Deciding bisimilarity between given and constructed BPA

- Algorithm for normed BPA (e.g. [Lasota, Rytter, 2006] working in $O(n^8 polylog\ n)$) can be used
- We propose a specialized algorithm
- It is based on ideas from algorithms deciding bisimilarity between BPA and finite state systems (e.g. Kučera, Mayr, 2002)
- It uses the fact that constructed BPA is almost a finite state system
- Our algorithm seems to have better complexity in this particular case, but we provide no analysis in this paper

Algorithm recapitulation

- Transform (M_0, Δ) to bisimilar (M_0', Δ') in the prime form
- Check three conditions, possibly end with answer $\alpha_0 \nsim M_0$
- Construct reachability graph of M_0' - markings with all tokens in one SF-place are "frozen"
- If the number of unfrozen markings exceeds $4N^2$ end with answer $\alpha_0 \nsim M_0'$
- Construct BPA Σ' with initial configuration α_0'
- Check whether $\alpha_0 \sim \alpha_0'$

All steps of this algorithm are polynomial hence the problem nBPA-nBPP-bisim is polynomial.

Thank you