Mathematical Logic

Exam test
December 18th, 2013

Name: **Surname:** **Login:**

1. Find some ***models of the following formulas***:
   $\forall x\,[A(x) \supset B(x)]$
   $\exists x\,[A(x) \wedge B(x)]$
   $\forall x\,P(x, f(x))$

2. ***Prove the logical validity of the formula***, using
   (a) resolution method, (**b**) **natural deduction**

   $[\forall x\,(P(x) \supset Q(x)) \wedge \exists x\,\neg Q(x)] \supset \exists x\,\neg P(x)$

3. ***Using any method, prove the validity of the following argument***:

   Tom is at home or went shopping.
   If Tom went shopping then he bought some milk.
   Tom didn't buy any milk.
   _____
   Tom is at home.

4. **Define by a formula of FOL the following set-theoretical relations:**
   A is a subset of *B* ($A \subseteq B$)
   A is a subset of the complement of *C* with respect to *B* ($A \subseteq B/C$)

5. **Formalise in the FOL language** the following propositions:
   a) The sets *A* and *B* have a non-empty intersection. ("Some *A*s are *B*s".)
   b) All numbers are odd or even.
   c) The set *A* is a *proper* subset of the set *B.*
   d) No *A* is *B.*
   e) Some *A*s are not *B*s.

6. Write down in **PROLOG**:
   Every student is younger than Tom's mother.
   Jack and Peter are students.
   Who is younger than Tom's mother?

   How will your Prolog program answer?
   Will the program answer that Tom is younger than his mother?