

Task No 5.

1. Prove by *natural deduction* that the following formulas are tautologies:

$$(\neg p \vee q) \supset (p \supset q)$$

$$[(p \vee q) \wedge (q \supset r)] \supset (p \vee r)$$

$$\exists x \forall y P(x,y) \supset \forall y \exists x P(x,y)$$

$$\exists x [P(x) \wedge Q(x)] \supset [\exists x P(x) \wedge \exists x Q(x)]$$

$$[\forall x P(x) \vee \forall x Q(x)] \supset \forall x [P(x) \vee Q(x)]$$

$$\exists x P(x) \supset (\forall x [P(x) \supset Q(x)] \supset \exists x Q(x))$$

2. Prove by *natural deduction* that the following arguments are valid:

We have sunshine or stay at home.

If we have sunshine then we are going to the mountains.

We did not go to the mountains.

We stayed at home.

Everybody loves somebody.

All those who are loved by Dracula die soon.

Somebody dies soon.

Everybody who admires the US president and votes for democracy, does not like Bin Laden.

Some do like Bin Laden though they vote for democracy.

Some vote for democracy but do not admire the US president.