Exercise *(to Lecture 9)*

1) Analyse the sentence "Dividing 5 by 0 is improper and Tilman knows *it*, while John doesn't believe *it* because *he (John)* believes that 5:0 = 1"
2) Prove that this argument is valid:

Dividing 5 by 0 is improper and Tilman knows *it*,
while John doesn't believe *it* because *he* believes that 5:0 = 1

There is a construction such that Tilman knows that ***it*** is improper while John believes ***it*** produces 1

Ad (1)

Types. *Div*/(τττ); 0, 1, 5/τ; *Improper*/(ο$*_n$): the class of constructions *v*-improper for every valuation *v*; *Tilman*, *John*/ι; *Know*, *Believe*/$(\text{ο}\iota *_n)_{\tau\omega}$: hyperintensional attitudes to a construction of a truth-value; *it* → $*_n$: anaphoric variable; *he* → ι: anaphoric variable.

Synthesis and type checking.

First clause.

$[^0Div\ ^05\ ^00]/*_1 \to \tau$; $[^0Improper\ ^0[^0Div\ ^05\ ^00]]/*_2 \to o$;

Second and third clause.

$[[^0Know_{wt}\ ^0Tilman\ it] \wedge \neg[^0Believe_{wt}\ ^0John\ it]] \to o$: open construction that is typed to *v*-construct a truth-value according as Tilman knows it and John doesn't believe it. We have to complete it by substituting the subject of Tilman's and John's attitude, i.e. the construction $[^0Improper\ ^0[^0Div\ ^05\ ^00]]$ for *it.* Here is how.

$^2[^0Sub\ [^0Tr\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]]\ ^0it\ ^0[[^0Know_{wt}\ ^0Tilman\ it] \wedge \neg[^0Believe_{wt}\ ^0John\ it]]] \to o$

According to the definition of the function *Sub*, and by applying the rule $^{20}C = C$, for any construction *C*, this construction is equivalent to (=)

$^{20}[[^0Know_{wt}\ ^0Tilman\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]] \wedge \neg[^0Believe_{wt}\ ^0John\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]]] =$

$[[^0Know_{wt}\ ^0Tilman\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]] \wedge \neg[^0Believe_{wt}\ ^0John\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]]]$

Fourth clause.

$[^0Believe_{wt}\ he\ ^0[[^0Div\ ^05\ ^00] = ^01]] \to o$: open construction that is typed to *v*-construct a truth-value. We complete it by substituting ^{0}John for *he*.

$^2[^0Sub\ [^0Tr\ ^0John]\ ^0he\ ^0[^0Believe_{wt}\ he\ ^0[[^0Div\ ^05\ ^00] = ^01]]] =$

$^{20}[^0Believe_{wt}\ ^0John\ ^0[[^0Div\ ^05\ ^00] =_\tau ^01]] =_o [^0Believe_{wt}\ ^0John\ ^0[[^0Div\ ^05\ ^00] =_\tau ^01]]$

The analysis of the whole sentence comes down to this construction.

$$\lambda w\lambda t\ [[^0Improper\ ^0[^0Div\ ^05\ ^00]] \wedge$$
$$^2[^0Sub\ [^0Tr\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]]\ ^0it\ ^0[[^0Know_{wt}\ ^0Tilman\ it] \wedge \neg[^0Believe_{wt}\ ^0John\ it]]] \wedge$$
$$^2[^0Sub\ [^0Tr\ ^0John]\ ^0he\ ^0[^0Believe_{wt}\ he\ ^0[[^0Div\ ^05\ ^00] = ^01]]]] \to o_{\tau\omega}$$

Ad (2) ***proof***

In every possible world *w* and time *t* of evaluation, the following steps are truth-preserving:

1. $[[^0Improper\ ^0[^0Div\ ^05\ ^00]] \wedge$
$^2[^0Sub\ [^0Tr\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]]\ ^0it\ ^0[[^0Know_{wt}\ ^0Tilman\ it] \wedge \neg[^0Believe_{wt}\ ^0John\ it]]] \wedge$
$^2[^0Sub\ [^0Tr\ ^0John]\ ^0he\ ^0[^0Believe_{wt}\ he\ ^0[[^0Div\ ^05\ ^00] = ^01]]]]$
assumption
2. $[[^0Improper\ ^0[^0Div\ ^05\ ^00]] \wedge$
$[^0Know_{wt}\ ^0Tilman\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]] \wedge$
$[^0Believe_{wt}\ ^0John\ ^0[[^0Div\ ^05\ ^00] = ^01]] \wedge$
$\neg[^0Believe_{wt}\ ^0John\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]]]$
(1), def. of *Sub*, commutativity of $\wedge$
3. $[[^0Improper\ ^0[^0Div\ ^05\ ^00]] \wedge$
$[\lambda c\ ^2[^0Sub\ [^0Tr\ c]\ ^0it\ ^0[[^0Know_{wt}\ ^0Tilman\ ^0[^0Improper\ it]] \wedge$
$[^0Believe_{wt}\ ^0John\ ^0[^2it = ^01]]]]\ ^0[^0Div\ ^05\ ^00]] \wedge$
$\neg[^0Believe_{wt}\ ^0John\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]]]$
$c, it \rightarrow *_n;\ ^2it \rightarrow \tau;$ λ-abstraction, (2)
4. $[[^0Improper\ ^0[^0Div\ ^05\ ^00]] \wedge$
$[^0\exists\ \lambda c\ ^2[^0Sub\ [^0Tr\ c]\ ^0it\ ^0[[^0Know_{wt}\ ^0Tilman\ ^0[^0Improper\ it]] \wedge$
$[^0Believe_{wt}\ ^0John\ ^0[^2it = ^01]]]]] \wedge$
$\neg[^0Believe_{wt}\ ^0John\ ^0[^0Improper\ ^0[^0Div\ ^05\ ^00]]]]$
I$\exists$, (3)
5. $[^0\exists\ \lambda c\ ^2[^0Sub\ [^0Tr\ c]\ ^0it\ ^0[[^0Know_{wt}\ ^0Tilman\ ^0[^0Improper\ it]] \wedge$
$[^0Believe_{wt}\ ^0John\ ^0[^2it = ^01]]]]]$ E$\wedge$, (4)

Gloss. Indeed, in the step (4) we can introduce $\exists$-quantifier, because the class of constructions produced by $\lambda c\ ^2[^0Sub\ [^0Tr\ c]\ ^0it\ ^0[[^0Know_{wt}\ ^0Tilman\ ^0[^0Improper\ it]] \wedge [^0Believe_{wt}\ ^0John\ ^0[^2it = ^01]]]]$ is non-empty; according to (3) it contains the construction $[^0Div\ ^05\ ^00]$.

Remark. The consequent of the argument, namely the construction (5) is entailed, provided John is able to apply the above rule $^{20}C = C$, which we assume.