Properties on the edge

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1. Introduction

In April 2007, after a talk at TU Delft, when discussing the TIL conception of logical analysis of natural language, Maarten Franssen put this question to us: “Do wharrots exist?”. At first this seemed a futile, unreasonable question. However, it gave rise to a fruitful and rich discussion. The question concerns bare individuals, mereological sums, and the way of ascribing properties to individuals; and the answer reaches far into the core of TIL. Of course, at first you want to ask, “What do you mean by ‘wharrot’?” Reply: “Well, a ‘pair-individual’ consisting of two other individuals, one with the property of being a whale and the other with the property of being a carrot.” Rejoinder: “Nonsense!” Surprisingly, though, a moment’s reflection reveals that, of course, such an individual exists; indeed, they are plentiful.

TIL operates with a constant domain of individuals for all worlds and times. Thus there is no room for *possible individuals*, and individuals trivially exist. Over the universe of individuals an infinite hierarchy of partial functions is examined, among which some of the most important are *intensions*. Intensions are partial functions with domain in *possible worlds* and *times*. Hence what varies are the values that (non-trivial) intensions have in different worlds (and at different times), and not the domains that different worlds and times have on varying-domain theories. TIL also rejects individual essentialism; no individual bears any non-trivial property by any sort of logical necessity. This is not to say, though, that we reject essentialism across the board; far from it. We have built up an essentialist theory, according to which relations of logico-conceptual necessity obtain between various kinds of intension. The result is *intensional essentialism*, which says, roughly, that, necessarily, if x is a/the F then x is also a/the G, because being a/the G is in the essence of being a/the F.

There is, however, a frequently voiced objection to individual anti-essentialism: If, for instance, my car is disassembled into its elementary parts, then my car no longer exists. Hence, being a car is essential of the individual referred to by ‘my car’. Disregarding the problem of indexicals and letting ‘my car’ mean, for instance, Marie’s only car, our answer is this. First, what is *denoted* by ‘Marie’s car’ is not an individual, but an individual office (or role), which is an intension having different individuals as values in different possible worlds at different times. Actually and currently my car is a Susuki Swift 3T45722, but it used to be Škoda Felicia Combi, and there is no logical necessity connected with the fact that I bought a Susuki Swift, for I could have bought any other car (even of the same brand). Second, the individual currently referred to as ‘my car’, or ‘3T45722’, does not cease to exist even when being taken apart into its most elementary parts. It just loses some properties, among them the property of being a car, the property of being composed of its current parts, etc, and acquires some other properties. If by chance somebody happened to assemble the parts together so that the individual would again acquire the property of being a car, I’d have no right to claim that was *my* individual car,[1] if the individual no more existed. Instead, when being disassembled, *my* individual unfortunately obtained the property of being completely disassembled and dissolved into elementary parts.

[1] As Tichý argues in (1987), where he uses the example of a watch being ‘repaired’ by a watchmaker so that it becomes a key.

Above we tacitly distinguished between *denoting* and *referring.* In TIL we have the means to spell out this difference, which concerns empirical expressions. The denotation is the same for all worlds and times, so that expressions with a complete meaning (i.e., roughly speaking, without indexical and anaphoric references) qualify as rigid designators with respect to denotation, because empirical expressions denote *intensions*. What varies is the reference; non-constant intensions do not return the same values at all worlds and times. But the reference relation is factual, *a posteriori* and extra-semantic. This is unlike the denotation relation, which is (conventionally) *a priori* and intra-semantic. The viability of the thesis that empirical terms and expressions denote intensions presupposes that we possess of a means to obtain an extension from an intension. For surely we do not want to end up claiming that the sentence, "Marie's car is a small vehicle" ascribes the property of being a small vehicle to the intension *Marie's car*. In TIL, functional *application* of intensions fits the bill. Thus there is no need for an operation/operator earmarked specially for extensionalization or predication. The logical analysis of, "Marie's car is a small vehicle" will contain several instances of functional application; apart from extensionalisation of particular intensions, the application from *Marie's car* to an individual, from *being a small vehicle* to a class of individuals, while the third is an application of the latter to the former (as classes are characteristic functions).

In this paper I am going to deal with the problem of bare individuals, their properties, intensional essentialism concerning intensions in general and properties in particular, and how bare individuals may consist of other bare individuals. The paper is organised as follows. First, in Section 2 I briefly introduce the basic principles of the TIL conception of logical analysis of natural language. In Section 3 I deal with intensional essentialism, existence and the predication of non-trivial properties of individuals. In Section 4 I deal with Cmorej's view that (partly) trivial properties are had by some individuals necessarily. Finally, in Section 5 I examine Cmorej's objections against individual anti-essentialism as presented in his (1988) concerning mereological sums.[2]

2. Epistemic framework and logical foundations of TIL

The informal, pre-theoretical base of logical analysis of natural language (LANL), as construed by TIL, is thoroughly explained in Tichý (1988, pp. 177-200). Here we briefly summarise the main pre-theoretical notions that are necessary for building the theory.

First of all, the main methodological principle of TIL-based LANL is this:

> To explicate a system of intuitive, pre-theoretical, notions is to assign to them, as surrogates, members of the functional hierarchy over a definite objectual base. Relations between the intuitive notions are then represented by the mathematically rigorous relationships between the functional surrogates.
>
> Tichý (1988, pp.194-195)

To account for the expressive power of a given language shared by a community of language-users, Tichý introduces the concept of *epistemic framework* and the concepts of intensional and objectual base affiliated with it. The intensional base contains 'intuitively, pre-theoretically given determiners' that are no further definable, like 'colours, heights, propositional attitudes and the like'; see Tichý (1988, p. 199)). The intensions defined over an objectual base attempt to capture them theoretically. They do so by means of assignments to the functions defined over the objectual base. Tichý calls the totality of these assignments an *explication* of the intensional base by means of the objectual base. An epistemic framework is then an intensional base garnished with an explication.

[2] Some portions of this paper reproduce paragraphs taken from Duzi, Jespersen and Materna (Ms).

The pre-theoretically understood elements of the objectual base *B* may in principle be pretty much whatever. But for the purposes of natural-language analysis, it has turned out that the elements must include, at least, truth-values, individuals, times, and possible worlds. Formally, $B = \{o, \iota, \tau, \omega\}$, each element of which is a non-empty set and disjoint from any other of the three sets. Objects of these four kinds are all non-functions (or, functions of zero arity, if you like), and cannot be defined (though characterised) within TIL. They are, in a word, logically primitive relative to *B*. However, the functions arising from *B* by combining elements drawn from it can be defined within TIL; this is required if we wish to display functional dependencies in accordance with our functional approach. The objectual base *B*, for its part, can be thought of as being among the fundamental ontological assumptions — or 'ontological commitments', as some would have it — of TIL.

A most important part of the explication is the interpretation of possible worlds. It goes as follows:

> By an *intension/time* I shall understand an ordered couple consisting of a member of intensional base and a moment of time. A *determination system* is then an assignment which assigns to (some) intension/times unique objects over $\{\iota, o, \tau, \omega\}$ in such a way that if the type corresponding to the intension is $\xi_{\tau\omega}$[3] then the object assigned to the intension/time is ξ. Briefly, a determination system specifies one combinatorial possibility as to what objects are determined…by what intensions at what times.
>
> Now to interpret the basic category ω is to assign to each of its members a unique determination system.
>
> Tichý (1988, p.199)

In what follows the *epistemic framework* of TIL is described.

Universe of discourse: ι. The members of the universe are *individuals*. The individuals are *bare individuals*. This means that all properties possessed by an individual necessarily are in some sense trivial. In Section 4 I will explain in which sense some properties are trivial. Roughly, trivial properties are either *constant* functions (i.e., properties that have a constant extension — a set of individuals — as value in all possible worlds and times), or non-constant functions with a constant subset of their possible extensions. All non-trivial properties are possessed by an individual only contingently. A bare individual is, then, what remains if one abstracts from all its non-trivial properties. Another important feature of the universe is that it is one in number; there are no other universes in other possible worlds, so there are no *possibilia* ('possible individuals').

Truth-values: o. There are just two truth-values (in this sense TIL is classical logic). Any abstract objects can serve as surrogates, but we have to interpret them and say that **T** is the truth-value *True* and **F** the truth-value *False.*

Times or *real numbers*: τ. The easy interpretation is described in Tichý (1988, p.199); choosing the origin 0 of the time scale and a specific duration of time between 0 and the time represented by 1, we get the result that every real number will represent a unique moment of time and *vice versa.*

Possible worlds: ω. The collection of primitive determiners makes up the *intensional base* (relative to a given language). Every member of the intensional base conjugated with a moment of time determines some object. A *possible world* is interpreted as specifying "one

[3] See below; it is the type of a function $(\omega \rightarrow (\tau \rightarrow \xi))$, denoted $((\xi\tau)\omega)$, or shortly $\xi_{\tau\omega}$, for a type ξ.

combinatorial possibility as to what objects are determined… by what intensions (i.e., members of the intensional base) at what times" (Tichý 1988, p.199).

The way of construing possible worlds described above is *Tractarian* in that it takes possible worlds as collections of *states-of-affairs* rather than of objects. In this respect we could speak of 'the maximum consistent set of (chronologies of) facts' as possible worlds are commonly understood. Also Hintikka seems to accept this conception, but his possible worlds are *epistemic*, dependent on particular language-users; see, e.g., Hintikka and Hintikka (1989).

The semantics of TIL is not set-theoretical but *procedural.* The meaning of an expression *E* is not the object denoted by *E* (if any), but the procedure expressed by *E* (known as a TIL *construction*) that yields the denoted object as its output, or in well-defined cases fails to yield anything. *Qua* procedures, constructions are algorithmically structured, unlike set-theoretical objects, which lack any structure. *Qua* abstract, extra-linguistic entities, constructions are reachable only *via* a verbal definition: the 'language of constructions' is a modified hyperintensional version of the typed λ-calculus, where Montague-like λ-terms denote, not the functions constructed, but the constructions themselves. Constructions operate on input objects (of any type, even on constructions) and yield as output objects of any type. In this way constructions construct *partial functions*; and functions, rather than relations or sets, are basic objects of our ontology.

By claiming that constructions are algorithmically structured, we mean the following. A construction *C* consists of particular steps, or *constituents,* that have to be executed in order to execute *C.* But there is no way of executing the lowest-level, non-constructional objects. A constituent of a construction has always to be another construction. Thus non-constructional objects have to be supplied as input for a construction to operate on by atomic constructions. A construction is atomic if it is a procedure that does not contain any other construction as a used subconstruction (a 'constituent').[4] There are two atomic constructions that supply objects (of any type) on which complex constructions operate: *Variables* and *Trivializations.* The other constructions are *compound*, as they consist of other constructions. They are *Composition*, *Closure*, *Execution* and *Double Execution.*

TIL constructions, as well as the entities they construct, all receive a type. The definitions proceed inductively. First, we define types of order 1 (simple types); second, constructions operating on types; finally, the whole ontology of entities organised into a ramified hierarchy of types.

Definition 1 (*Types of order 1*)
Let *B* be a base, i.e., a collection of pair-wise disjoint, non-empty sets.

i) Every member of *B* is an elementary *type of order 1 over B.*

ii) Let α, β_1, ..., β_m ($m > 0$) be types of order 1 over *B*. Then the collection $(\alpha\beta_1... \beta_m)$ of all *m*-ary partial mappings from $\beta_1 \times ... \times \beta_m$ into α is a functional *type of* order *1 over B.*

iii) Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii). □

Remark. For the purposes of natural-language analysis we choose the *objectual base* described above. Thus, the objectual base *B* consists of the following atomic types:

ο, the set of truth-values **T, F**;

ι, the set of individuals, i.e., the universe of discourse;

[4] For the use-mention distinction and definition of a constituent, see Duží (2007).

τ, the set of real numbers or moments of time;

ω, the set of possible worlds, i.e., the logical space.

TIL is an open-ended system. The above objectual base {ο, ι, τ, ω} was chosen, because it is apt for natural-language analysis, but in the case of mathematics a (partially) distinct base would be appropriate; for instance, the base consisting of natural numbers, of type ν, and truth-values. The derived types would then be defined over {ν, ο}.

Definition 2 (*Construction*)

i) *Variables x, y, z, ...*are ***constructions*** that construct objects of the respective types dependently on valuations *v*. To each type α countably many variables are assigned, and elements of α can be arranged into sequences. Let a total valuation function *v* be given that associates variables $x_1, x_2, \ldots, x_n, \ldots$ with a sequence *Seq* of objects $a_0, a_1, \ldots, a_n, \ldots$ of type α. Then the *variable* x_n *v(aluation)-constructs* the n^{th} object a_n of *Seq* relative to *v*.

ii) *Trivialization*: Where X is an object whatsoever (an extension, an intension or a *construction*), ^{0}X is a ***construction*** called *Trivialization.* It constructs X without any change.

iii) *Composition* $[X\ Y_1 \ldots Y_m]$ is a ***construction***: If *X v-constructs* a function *f* of a type $(\alpha\ \beta_1 \ldots \beta_m)$, and $Y_1,\ldots,Y_m$ *v-construct* entities $B_1,\ldots,B_m$ of types $\beta_1,\ldots,\beta_m$, respectively, then the *Composition* $[X\ Y_1 \ldots Y_m]$ *v-constructs* the value (an entity, if any, of type α) of *f* on the argument $\langle B_1, \ldots, B_n\rangle$. Otherwise the *Composition* $[X\ Y_1 \ldots Y_m]$ does not *v-construct* anything: it is *v-improper.*

iv) *Closure*: Let $x_1, x_2, \ldots, x_m$ be pairwise distinct variables and *Y* a construction. Then $[\lambda x_1 \ldots x_m\ Y]$ is a ***construction*** called *λ-Closure* (or simply *Closure*). It *v-constructs* the following function *f.* Let $v(B_1/x_1,\ldots,B_m/x_m)$ be a valuation identical with *v* at least up to assigning objects $B_1,\ldots,B_m$ to variables $x_1,\ldots x_m$. If *Y* is $v(B_1/x_1,\ldots,B_m/x_m)$-improper (see iii), then *f* is undefined on $\langle B_1,\ldots,B_m\rangle$. Otherwise the value of *f* on $\langle B_1,\ldots,B_m\rangle$ is the object $v(B_1/x_1,\ldots,B_m/x_m)$-constructed by *Y.*

v) *Execution*: ${}^1X$ is a ***construction*** called *Execution* that *v*-constructs the entity (if any) *v*-constructed by *X.* Otherwise the *Execution* ${}^1X$ is *v*-improper.

vi) *Double Execution*: ${}^2X$ is a ***construction*** called *Double Execution.* It *v-constructs* the entity (if any) *v-constructed* by the construction *X'* *v*-constructed by *X.* Otherwise the *Double Execution* ${}^2X$ is *v-improper*.

vii) Nothing is a ***construction***, unless it so follows from (i) through (vi). □

Remark. We use the terms 'mapping' and 'function' synonymously. By 'partial mapping' we mean a mapping that associates every argument (of the respective type) with at most one value (of the respective type); a total function is then a special kind of the former, namely a mapping that associates every argument with just one value.

Remark. Outer brackets will be omitted whenever no confusion can arise.

Constructions that construct entities of order 1 are constructions of order 1. They belong to a type of order 2, denoted by $*_1$. The type $*_1$ serves as a base for the induction rule: any collection of partial functions, type $(\alpha\ \beta_1\ldots\beta_n)$, involving $*_1$ in their domain or range is a *type of order 2*. Constructions belonging to a type $*_2$ that identify entities of order 1 or 2, and partial functions involving such constructions, belong to a *type of order 3*; and so on *ad infinitum*.

Definition 3 (*Ramified hierarchy of types*)
Let B be a base (a collection of pair-wise disjoint, non-empty sets).

T_1 *(types of order 1)*: defined by Definition 1.

C_n *(constructions of order n)*

i) Let x be a variable ranging over a type of order n. Then x is a *construction of order n over B*.
ii) Let X be a member of a type of order n. Then ${}^0X$, ${}^1X$, ${}^2X$ are *constructions of order n over B*.
iii) Let $X, X_1,\ldots,X_m$ ($m > 0$) be constructions of order n over B. Then $[X\ X_1\ldots X_m]$ is a *construction of order n over B*.
iv) Let $x_1,\ldots x_m, X$ ($m > 0$) be constructions of order n over B. Then $[\lambda x_1\ldots x_m\ X]$ is a *construction of order n over B*.
v) Nothing is a construction of order n over B unless it so follows from $\mathbf{C}_n$ (i)-(iv).

T_{n+1} *(types of order n + 1)*
Let $*_n$ be the collection of all constructions of order n over B.

i) $*_n$ and every type of order n are *types of order* $n + 1$.
ii) If $0 < m$ and $\alpha, \beta_1,\ldots,\beta_m$ are types of order $n + 1$ over B, then $(\alpha\ \beta_1\ \ldots\ \beta_m)$ (see T_1 ii)) is a *type of order* $n + 1$ *over B*.
iii) Nothing is a *type of order n + 1 over B* unless it so follows from (i) and (ii). □

Definition 4 (*quantifiers ∀ and ∃*)
The universal quantifier $\forall^\alpha$ and the existential quantifier $\exists^\alpha$ are functions of type $(o(o\alpha))$ defined as follows. Let x v-construct elements of type α, A v-construct elements of type o. Then

- the Composition $[{}^0\forall^\alpha\ [\lambda x\ A]]$ v-constructs the truth-value **T**, iff the Closure $[\lambda x\ A]$ v-constructs the whole type α, otherwise **F**;
- the Composition $[{}^0\exists^\alpha\ [\lambda x\ A]]$ v-constructs **T**, iff the Closure $[\lambda x\ A]$ v-constructs a non-empty class of elements of type α, otherwise **F**. □

Notational conventions. An object X of a type α is called an α-object, denoted 'X/α'. That a construction C v-constructs an α-object will be denoted '$C \rightarrow_v \alpha$'. If a construction C v-constructs an α-object a entirely independently of valuation, we say that C constructs a and write '$C \rightarrow \alpha$'. We shall often write '$\forall x\ A$', '$\exists x\ A$' instead of '$[{}^0\forall^\alpha\ \lambda x\ A]$', '$[{}^0\exists^\alpha\ \lambda x\ A]$', respectively, when no confusion can arise. We shall also often use infix notation without Trivialization when using constructions of the truth-functions $\wedge$ (conjunction), $\vee$ (disjunction), $\supset$ (implication), $\equiv$ (equivalence) and negation ($\neg$), and when using a construction of an identity relation.

Intensions are functions of type $(\beta\omega)$, β frequently the type of an α-chronology: $(\alpha\tau)$. Thus an α-intension is of type $((\alpha\tau)\omega)$, which we abbreviate by '$\alpha_{\tau\omega}$'. When X is a construction of an α-intension, $X \rightarrow \alpha_{\tau\omega}$, the frequently used construction $[[X\ w]\ t]$ — the intensional descent of the intension v-constructed by X — is abbreviated by 'X_{wt}'.

Some important kinds of intensions are:

Propositions/$o_{\tau\omega}$. They are denoted by empirical (declarative) sentences.

Properties of members of a type α, or simply *α-properties*/$(o\alpha)_{\tau\omega}$.[5] General terms (some substantives, intransitive verbs, adjectives denoting properties, mostly of individuals).

Relations-in-intension/$(o\beta_1\ldots\beta_m)_{\tau\omega}$. For example, transitive empirical verbs and attitudinal verbs denote such relations. Omitting $_{\tau\omega}$ we get the type $(o\beta_1\ldots\beta_m)$ of *relations-in-extension* (to be found mainly in mathematics and logic).

α-roles/*α-offices*/$\alpha_{\tau\omega}$, $\alpha \neq (o\beta)$, frequently $\iota_{\tau\omega}$, often denoted by the concatenation of a superlative and a noun ('the highest mountain').

3. Intensional essentialism

Intensional essentialism comes in handy, for instance, when spelling out the *de dicto*/*re* ambiguities besetting, e.g., "Necessarily, Marie's car is a car". Taken *de dicto*, it is true, for there is a necessary, *a priori* link between the intensions *Marie's car* and *being a car* — you cannot have the former without also having the latter. Taken *de re*, it is false, for nothing of a logical or conceptual nature forces whatever individual is Marie's car to be a car. It is neither true nor false, if Marie happens to have no car, for then there is nothing of which it would be either true or false that *it* is a car. The leading idea is that modality *de dicto* is based on *a priori* relations between intensions, while modality *de re* is based on bare particulars.

In what follows we explain how two intensions may be conceptually related so that having one necessitates having the other as well. When there is necessitation of this kind, we say that one intension is *essential* of the other, and the two intensions are related by the *Requisite* relation. It is intensions, and not extensions such as individuals, that are the bearers of essential properties. Instead our individuals are 'bare' in the sense that no non-trivial intension is necessarily true of them.

In general, intensions of any type can be related by the Requisite relation. For instance, if *a* found the murderer of *b* after a preceding search, then the murderer must exist. Thus the existence of the murderer is essential for a successful search, i.e., a case of finding. But *Exist(ence)* is not a non-trivial property of individuals. All individuals trivially exist. Instead, it is a property of *intensions*, which translates into the property of being instantiated (in the case of properties) or the property of being occupied (in the case of offices). In our case *Exist* is a property of an individual office. The Requisite relation between finding and existence is defined as follows:

$$[{}^0Req^f\ {}^0Exist\ {}^0Being\_found] = \forall w \forall t\ [\forall u\ [[{}^0Being\_found_{wt}\ u] \supset [{}^0Exist_{wt}\ u]]].$$

Types: $Req^f/(o(o\iota_{\tau\omega})_{\tau\omega}(o\iota_{\tau\omega})_{\tau\omega})$; $Being\_found$, $Exist/(o\iota_{\tau\omega})_{\tau\omega}$; $u \rightarrow \iota_{\tau\omega}$.

Now we may define the property of an office *Being_found* as follows:

$${}^0Being\_found = \lambda w \lambda t\ \lambda u\ [\exists x\ [{}^0Find_{wt}\ x\ u]],$$

$Find/(o\iota\iota_{\tau\omega})_{\tau\omega}$; $x \rightarrow \iota$.

Substituting the right-hand side for the left-hand one to the above definition, we obtain:

$$\forall w \forall t\ [\forall u\ [[[\exists x\ [{}^0Find_{wt}\ x\ u]] \supset [{}^0Exist_{wt}\ u]]] =$$

$$\forall w \forall t\ [\forall u \forall x\ [[{}^0Find_{wt}\ x\ u] \supset [{}^0Exist_{wt}\ u]]].$$

In particular, necessarily, if *a* found the murderer of *b* then the murderer exists:

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[5] Collections, sets, classes of α-objects are members of type $(o\alpha)$; TIL handles classes (subsets of a type) as characteristic functions. Similarly, relations (-in-extension) are of type(s) $(o\beta_1\ldots\beta_m)$.

$$\forall w \forall t\ [[Find_{wt}\ {}^0a\ \lambda w \lambda t\ [{}^0Murderer\_of_{wt}\ {}^0b]] \supset [{}^0Exist_{wt}\ \lambda w \lambda t\ [{}^0Murderer\_of_{wt}\ {}^0b]]].$$

Additional types: $Murderer\_of/(\iota\iota)_{\tau\omega}$; $a,b/\iota$.

Remark: The existence of a murderer is, however, not a *presupposition* of finding one. If it were, then existence would be a Requisite of finding as well as *not*-finding, which is not the case.[6]

The most important kinds of Requisite relation are:

(1) $Req_1\ /(o\ (o\iota)_{\tau\omega}\ (o\iota)_{\tau\omega})$: an individual property being a requisite of another such property.

(2) $Req_2\ /(o\ (o\iota)_{\tau\omega}\ \iota_{\tau\omega})$: an individual property being a requisite of an individual office.

Whales being mammals is an example of the first kind of relation. Necessarily, *if* an individual *a* is a whale at some $\langle W,T \rangle$-pair *then a* is also a mammal at $\langle W,T \rangle$. It is an open question (epistemologically and ontologically speaking) *whether a* is a whale at $\langle W,T \rangle$. Establishing whether *a* is a whale requires investigation *a posteriori*. On the other hand, establishing whether *a* must be a mammal in case *a* happens to be a whale is *a priori*, the Requisite relation being in-extension and as such independent of what is true at any $\langle W,T \rangle$.

The second kind of Requisite obtains, for instance, between the property of being a king and the office of King of France. It is true at all *w*, *t* (including those where the office of King of France is vacant, like the actual one) that being a king is essential for an individual to be the King of France. The Requisite relation obtains for all worlds *w* and times *t*, and the values at *w, t* of particular intensions are irrelevant. Thus if an office *X* has the Requisite intension *Y*, this is so no matter whether *X* is occupied or vacant at a given $\langle W, T \rangle$.

Definition 5 (*Requisite relation between ι-properties*)
Let *X*, *Y* be constructions of intensions such that $X, Y \rightarrow (o\iota)_{\tau\omega}$; $x \rightarrow \iota$. Then

$$[{}^0Req_1\ Y\ X] = \forall w \forall t\ [\forall x\ [[{}^0True_{wt}\ \lambda w \lambda t\ [X_{wt}\ x]] \supset [{}^0True_{wt}\ \lambda w \lambda t\ [Y_{wt}\ x]]]]. \ \square$$

Gloss *definiendum* as, "*Y* is a requisite of *X*", and *definiens* as, "Necessarily, whatever *x* instantiates *X* at $\langle W,T \rangle$ also instantiates *Y* at $\langle W,T \rangle$."[7]

Definition 6 (*Requisite relation between a ι-property and a ι-office*)

Let *X*, *Y* be *constructions* of intensions such that $X \rightarrow \iota_{\tau\omega}$ and $Y \rightarrow (o\iota)_{\tau\omega}$.

Then $[{}^0Req_2\ \ Y\ X] = \forall w \forall t\ [[{}^0Exist_{wt}\ X] \supset [{}^0True_{wt}\ \lambda w \lambda t\ [Y_{wt}\ X_{wt}]]]$. $\square$

When defining the Requisite relation, partiality gives rise to the following complication both with respect to offices and properties. Let *X* be a construction of an ι-office, $X \rightarrow \iota_{\tau\omega}$, *Y* of a ι-property, $Y \rightarrow (o\iota)_{\tau\omega}$. If the Composition X_{wt} (corresponding to the intensional descent of the intension *v*-constructed by *X* in *w*, at *t*) is *v*-improper, it does not pick up any individual. Due to the semantic compositionality constraint informing TIL, no property can be truly predicated

[6] For details on notional attitudes of seeking and finding, see Duží *et al* (2007, §5.6.2), or Duží (2003).

[7] If a property *Y* is a requisite of a property *X*, then we also often say that *X* implies *Y*. In the computer science discipline of conceptual modelling, or, as it is fashionable to say today, of *ontology-building*, this relation gives rise to the so-called ISA-hierarchy classification and inheritance: individuals that instantiate the property *X* inherit all the attributes ascribed to them as being instances of *Y*. For instance, a whale inherits the attributes ascribed to mammals, like the production of milk in females for the nourishment of their young from mammary glands, or having endothermic, warm-blooded bodies, etc.

of an individual when there is none. So the Composition $[Y_{wt}\, X_{wt}]$ will be *v*-improper as well. The truth-functional connective of material implication ($\supset$/(ooo)) is such that when applied to a missing argument (a truth-value gap), the result is *v*-improper, making the Composition $[[{}^0Exist_{wt}\, X] \supset [Y_{wt}\, X_{wt}]]$ *v*-improper at those $\langle W,T\rangle$-pairs where X goes vacant. The whole *definiens* $\forall w \forall t\, [[{}^0Exist_{wt}\, X] \supset [Y_{wt}\;\; X_{wt}]]$ will, thus, construct False! A similar problem arises even in the case of properties. The reason is because properties are isomorphic to characteristic functions, and these functions can also have truth-value gaps. For instance, the property of having stopped smoking comes with a bulk of requisites like, e.g., the property of being an ex-smoker. Thus, the predication of such a property Y of an individual a may also fail, causing $[Y_{wt}\; {}^0a]$ to be *v*-improper. The remedy is easy, fortunately — just use the property of propositions of being true at $\langle W,T\rangle$: $True/(\text{o}\text{o}_{\tau\omega})_{\tau\omega}$.

Given a proposition P, $[{}^0True_{wt}\; {}^0P]$ *v*-constructs **T** if P is true at $\langle W,T\rangle$; otherwise (i.e., if P is false or else undefined at $\langle W,T\rangle$) **F**.

Remark. When defining a requisite of an *office* X, the antecedent condition on X being occupied is required. Otherwise we shall have the following invalid argument on our hands (see Tichý, 1979, pp. 408ff; 2004, pp. 360ff).

> *P* is a requisite of office *O*
>
> ---
>
> The occupant of *O* instantiates *P*.

This inference pattern is fallacious,

> for the premise may be true even if *O* is vacant, in which case the conclusion, so far from being true, is vacuous (i.e., lacks a truth value). (*Id.*, p. 408, p. 360, resp.)

However, a valid inference rule can be obtained by adding an extra premise to the effect that the relevant office is occupied:

> *P* is a requisite of office *O*
> Office *O* is occupied
>
> ---
>
> The occupant of *O* instantiates *P*.

Now we can define the *essence* of an α-intension X. It is a set of properties that characterises the intension X completely. For instance, if X is a ι-office, then necessarily any individual that occupies the office X must have any property belonging to E. Thus a general schema of an essence is this: let p, x be variables *v*-constructing α-properties and α-intensions, respectively. Then

$$ {}^0Essence = \lambda x \lambda p\, [{}^0Req\; p\; x]. $$

In particular, the essence of an individual property X and an individual office Y are defined as follows:

Definition 7 (*Essence of an individual property/office*)

Let $X/(\text{o}\iota)_{\tau\omega}$, $Y/\iota_{\tau\omega}$ be an individual property and office, respectively, and let $p \rightarrow (\text{o}\iota)_{\tau\omega}$ be a variable ranging over individual properties. Then the $Essence_1$ *of* X is the set of requisite properties of X, and the $Essence_2$ of Y is the set of requisite properties of Y:

$$ [{}^0Essence_1\, X] = \lambda p\, [{}^0Req_1\; p\; X], $$

$$ [{}^0Essence_2\, Y] = \lambda p\, [{}^0Req_2\; p\; Y]. \qquad \square $$

4. Bare individuals and essential properties

Above I argued for individual anti-essentialism: no individual has a non-trivial property by any sort of logical necessity. However, we did not define *non-trivial property* yet. Moreover, it remains at this point in time an open issue whether it is possible that a ι-object may lack all non-trivial properties in some possible world *W* at some time *T*. If it is, then such an individual will be 'bare' in a more dramatic sense than just not possessing any non-trivial properties necessarily (which is already considered dramatic enough in several quarters).

Consider an analysis of the property of individuals of having only trivial properties:

$$\lambda w \lambda t\ [\lambda x\ [\forall p\ [[p_{wt}\ x] \supset [^0 Triv\ \ p]]]].$$

Types: $\forall/(o(o(o\iota)_{\tau\omega}))$; $p \rightarrow (o\iota)_{\tau\omega}$; $x \rightarrow \iota$; $Triv/(o(o\iota)_{\tau\omega})$: the class of trivial ι-properties.

The analysis is a construction of a ι-property instantiated by individuals that do not have any non-trivial properties. The question is whether this ι-property does not go un-exemplified. The answer will depend on how restrictive or how liberal a notion of *non-trivial* ι-property is used; i.e., what the class *Triv* is taken to be. It certainly contains all *constant* properties, i.e., the properties that have a constant set of individuals as a value in all *w, t.* One of them is self-identity, which every individual necessarily possesses.

However, as Cmorej in (1988, 1966) pointed out, there are some non-constant properties that *some* individuals have necessarily. Consider Cambridge-like properties such as *being an x such that x is the same height as Pavel Tichý*. *Being the same height as Pavel Tichý* is not a constant function. Not all individuals have the same height as Pavel Tichý at all worlds and times, so the sets that are its extensions at various ⟨*W*, *T*⟩-pairs will not always have the same members. But whatever height Pavel Tichý may have at this or that ⟨*W*, *T*⟩, it is necessary that he should have exactly the same height as Pavel Tichý. The trick is to index a property to a specific individual *a*, such that, necessarily, *a* must have that property, without using a trivial property such as being self-identical.

So the intension *being the same height as Pavel Tichý* is insofar trivial. It is parasitic on the self-identity of Pavel Tichý, and has an *essential core*: namely, the set {Pavel Tichý}.[8] Similarly, the non-constant property *being the same age as a or b or c* has the essential core {*a*, *b*, *c*}. All individuals but *a*, *b* and *c* have this property contingently; only *a*, *b*, *c* have it necessarily. If the intension is non-trivial, its non-triviality is 'partial' or 'impure'; and if trivial, then its triviality is also impure.

Hence some non-constant properties can also be necessarily ascribed to *some* individuals (though *not to all* individuals), and are in some sense also trivial. Thus the characterisation of *non-trivial property* has to be extended.

The general direction in which to look for an answer is indicated by Tichý's distinction between *primary* and *parasitic* properties.

> A change in a thing clearly consists in the acquisition or loss of a property. But if any property is as good as any other, we get the odd result that a thing cannot change without every other thing changing as well. Suppose object *X* becomes red and consider another object, *Y*. *Y* will be spatially related to *X* in a definite way; suppose it is 50 miles due south from *X*. Then as *X* acquires redness, *Y* acquires the property of being 50 miles due south from a red object. This change in *Y*, however, is obviously a phoney change, because the property of being 50 miles due south from a red object is

[8] The term 'essential core' was coined by Pavel Cmorej in (1996). See also Cmorej (1988, 2006).

> a phoney, parasitic property. It is a property which will not figure in the specification of a possible world. To specify a possible world, one has to specify, *inter alia,* where each object is and what colour it is. Once all this has been fixed, there is no need to specify which objects have the property of being 50 miles due south from red objects; for all this has been implicitly specified already. While the extension of redness is part of what makes a world the world it is, the extension of the property of being 50 miles due to south from a red object is not. It is a parasitic property, a mere logical shadow cast by genuine — or, as we will say, *primary* — properties like being red and being at a certain place. For a thing to change, it must acquire or lose not any arbitrary property, but a primary one. We have seen that the possible worlds of a logical space are generated as distributions of the attributes in the intensional base through things. It is thus natural to identify primary properties, relations, etc. with those which correspond to the members of the intensional base.
>
> (Tichý, 1980, p. 271; 2004, p. 419.)

Every language is based on a definite universe of discourse (i.e., a collection of *individuals*) and an *intensional base*, which is the collection of *primary* intensions that the given language has predicates for. Hence primary properties are certainly *contingent* and *non-trivial.* No individual has a primary property of the intensional base necessarily, i.e., in all *w, t*.

Some of the derivative properties parasitic upon the *primary properties* are also contingent, like the above property of *being 50 km due south from a red object.* It is a contingent fact that an object *X* possesses at some time the property *being red.* This fact implies infinitely many facts where *derivative* properties play a role; for example, an object *Y* that happens to be 50 km due south from *X* gets the derivative property *being 50 km due south from a red object.* And *Y* does not have this property by a *logical* necessity. However, *Y* necessarily has the derivative property of *not* being 50 km due south from *itself*.[9]

Note that 'derivateveness' of a property does not concern a construction of the property. Any property can be constructed in infinitely many ways. Rather, it concerns necessary dependencies between the respective facts. For instance, the fact that an individual *a* is of this or that age is contingent. But there is a necessary correlation between the fact that, e.g, *a* is 50 and *a* is not younger than 30. There is no possibility that *a* were 50 and at the same time younger than 30. As explained in the previous Section 3, there are requisite relations between intensions. On the other hand, there are no such dependencies between primary properties of the intensional base; the respective basic facts are independent.

As explained above, non-constant properties with an essential core are *partly trivial*. They are *essential of some* individuals, namely of those belonging to the relevant essential core. All other individuals contingently have, or do not have, these properties. Hence if *P* is a partly trivial property, then there are at least two world/time pairs $\langle W, T\rangle$, $\langle W', T'\rangle$, such that P_{WT} is not the same set as $P_{W'T'}$. There is, however, a constant subset of the varying extensions of *P*, namely the essential core of *P*. Cmorej in (2006) calls these properties *partly essential*.

My hypothesis is that *partly trivial properties* with an essential core are parasitic on relations-in-intension such that, necessarily, the respective relation-in-extension (value in a world *w* and time *t*) is *reflexive*. The relations of being of the same height as some individuals, of the same age, of not being 20 years older, etc., can serve as examples. Of course, since *being the same age* is necessarily reflexive, an individual *a* cannot be of a different age as *a*, unless it would somehow, bizarrely, lose its identity.

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[9] We do not consider here subatomic particles of quantum physics, of course. After all, Heisenberg uncertainty principle has negligible effect on objects of macroscopic scale.

On the other hand, *purely trivial properties* are constant functions having the same set of individuals as value in all worlds w at all times t. Cmorej in (2006) calls these properties *essential*. Thus if P is a purely trivial property, the set P_{wt} is the same in all w, t, and it is the essential core of P. Every individual belonging to P_{wt} has P in all w, t, and every individual not belonging to P_{wt} lacks P in any w, t. The essential core of a purely trivial property P is either equal to the whole universe or it is a proper subset of the universe. Example of the former is the property of being self-identical, constructed by $\lambda w \lambda t\ \lambda x\ [x = x]$; examples of the latter are properties of being identical to a particular individual a, $\lambda w \lambda t\ \lambda x\ [x = a]$, being identical to an individual a or b, $\lambda w \lambda t\ \lambda x\ [[x = a] \vee [x = b]]$, being identical to neither a nor b, $\lambda w \lambda t\ \lambda x\ [\neg[x = a] \wedge \neg[x = b]]$, etc.

To sum up, a property P is trivial iff P has a constant essential core EC. Individuals belonging to EC have P necessarily, i.e., in all w at all t. In other words, the property P is essential of the elements of EC. Trivial properties are either purely trivial or partly trivial. The former are constant intensions and the latter non-constant. Our individual anti-essentialism thus qualifies as a 'modest one'. No individual has any non-trivial property necessarily. If an individual has a property p necessarily, then p is (purely or partly) trivial. Formally,

$$\forall p\ [\exists x\ \forall w \forall t\ [p_{wt}\ x] \supset [^0 Triv\ p]],$$

where $x \rightarrow \iota$, $p \rightarrow (o\iota)_{\tau\omega}$, $Triv/(o(o\iota)_{\tau\omega})$ — the class consisting of (purely or partly) trivial properties.[10]

A more outlandish property than *being the same height as Pavel Tichý* would be *being self-identical and the time is T* (for instance, noon on May 27, 2007). One of its constructions is ($T/(o\tau)$ being some fixed time interval)

$$\lambda w \lambda t\ \lambda x\ [[x = x] \wedge [^0 T\ t]].$$

(The construction $[^0 T\ t]$ suffices, because it is immaterial how the proposition that the time is 12 o'clock is constructed.) An individual satisfies this property if it is self-identical and the time is T (here, 12 o'clock) when it is tested for self-identity. The time is not always T, so the property is not trivial. But each x is, trivially, self-identical. Hence each individual has such properties at some times, and there are no purely bare individuals. However, as explained above, such a phoney property is parasitic on the self-identity of individuals.[11]

There is another criterion, according to which properties divide into *empirical* and *analytical.* Empirical property is such a property P that for *no* individual I it is possible to decide *a priori* whether P is, or is not, ascribed to I. It is always an *a posteriori* decision. On the other hand, the analytic property P is decidable *a priori* for all individuals. Obviously, purely trivial properties are analytic, and purely contingent properties are empirical. Partly trivial properties should be decidable analytically *a priori* of the individuals belonging to the essential core. Of course, we do not need any experience in order to decide whether an individual a is of the same age as an individual a or b. It is knowable *a priori.* However, as Cmorej pointed out in (1988), it is an open issue whether there are some properties that are partly trivial in a less obvious way, for which the respective essential core would be decidable only *a posteriori.*

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[10] It seems that Pavel Tichý, when formulating his thesis of individual anti-essentialism, did not consider partly trivial properties that are non-constant intensions yet essential of some individuals. Thus the idea of modest anti-essentialism is much due to Pavel Cmorej.

[11] These paragraphs arose from a discussion with Cmorej in the spring and summer of 2005 and also from Cmorej (1988, 1996). In particular the above definition of the non-constant property involving a particular time interval was proposed by Cmorej in (2006, p. 149) in order to prove that there are *no* bare individuals.

5. The part-whole relation and identity of individuals

At the outset of this paper I argued that a material entity that is a mereological sum of a number of parts, such as a particular car, is a simple, non-structured individual. Only its design, or *construction,* is a structured, hence complex procedure. Tichý says:

> [I]t would seem obvious that if the fountain pen is not in fact a complex then it is an error to *conceive* of it as one. It is undeniable, on the other hand, that when we say that the pen consists of the body and the cap, we do advert to something complex. What we are saying is that the pen can be *constructed* as a mereological sum of the body and the cap. It is this *construction* which is complex and uniquely decomposable into ultimate constituents, the body, the cap, and the mereological sum function. The discontinuous piece of plastic itself is a simple entity.
>
> By the same token, a *car* is a simple entity. But is this not a *reduction ad absurdum*? Are cars not complex, as anyone who has tried to fix one will readily testify?
>
> No, they are not. If a car were a complex then it would be legitimate to ask: Exactly how complex is it? Now how many parts does a car consist of? One plausible answer which may suggest itself is that it has three parts: an engine, a chassis, and a body. But an equally plausible answer can be given in terms of a much longer list: several spark plugs, several pistons, a starter, a carburettor, four tyres, two axles, six windows, etc. Despite being longer the latter list does not overlap with the former: neither the engine, nor the chassis nor the body appears on it. How can that be? How can an engine, for example, both be and not be a part of one and the very same car?
>
> There is no mystery, however. It is a common place that a car can be *decomposed* in several alternative ways. … Put is in other words, a car can be *constructed* in a very simple way as a mereological sum of three things, or in a more elaborate way as a mereological sum of a much larger set of things.
>
> [W]e undeniably are inclined to point to a car and say: This is a complex thing. But this only shows that we are not referring to the bare vehicle. It will be suggested that we are referring to the car *considered qua a mechanism to carry passengers from place to place.* This is how a mechanic might consider it. A physicist may consider it as a cloud of molecules or atoms, and a haulier as just a simple lump of cargo. But the manner in which someone considers it has no effect on the car itself. The truth of the matter is that our tendency to ascribe complexity to the car itself betrays confusion. What we are referring to is a conflation of an individual, the car, and a certain *design*, a definite way the car can be put together from certain components. The complexity we have in mind *pertains to the design*, *not to the car itself.* Otherwise we would have to maintain (as some indeed do) that the car considered by a mechanic is not the same car as the one considered by the physicist or the haulier.
>
> (1995, pp. 179-80)

Thus it is a contingent fact that this or that individual consists of other individuals creating a mereological sum. Actually, many, maybe indeed *all* concrete, physical individuals are such mereological sums, consisting of other individuals, and decomposable in many different ways.

Now we can try to answer the question posed at the outset as to whether wharrots exist. The first answer might be a positive one. Yes, there are many wharrots that are composed of a whale and a carrot. They are simple pairs, or couples, contingently having the property that the carrot-part is at this or that distance from its whale-part. Unless we specify a more complicated *way* of a wharrot *design*, or composition, they exist almost trivially.

Thus the property of being a wharrot, where ‘wharrot’ means simply a pair of a ‘whale-part’ and a ‘carrot-part’, is trivially instantiated. However, if there is another condition specified for an individual in order to be a wharrot, like, for instance, being *one* living creature consisting of a carrot and a whale, then wharrots thus understood cannot exist, because the DNA structure of a whale is incompatible with that of a carrot. Similarly, centaurs do not exist, because the DNA structure of a horse is incompatible with that of a human being.

The answer whether wharrots exist thus depends on the *meaning* of ‘wharrot’ in Franssen’s vernacular; in other words, which of the possible properties are assigned to ‘wharrot’ as its denotation. Here are four possible ‘wharrot definitions’, cf. the photo below taken during the discussion in Delft:

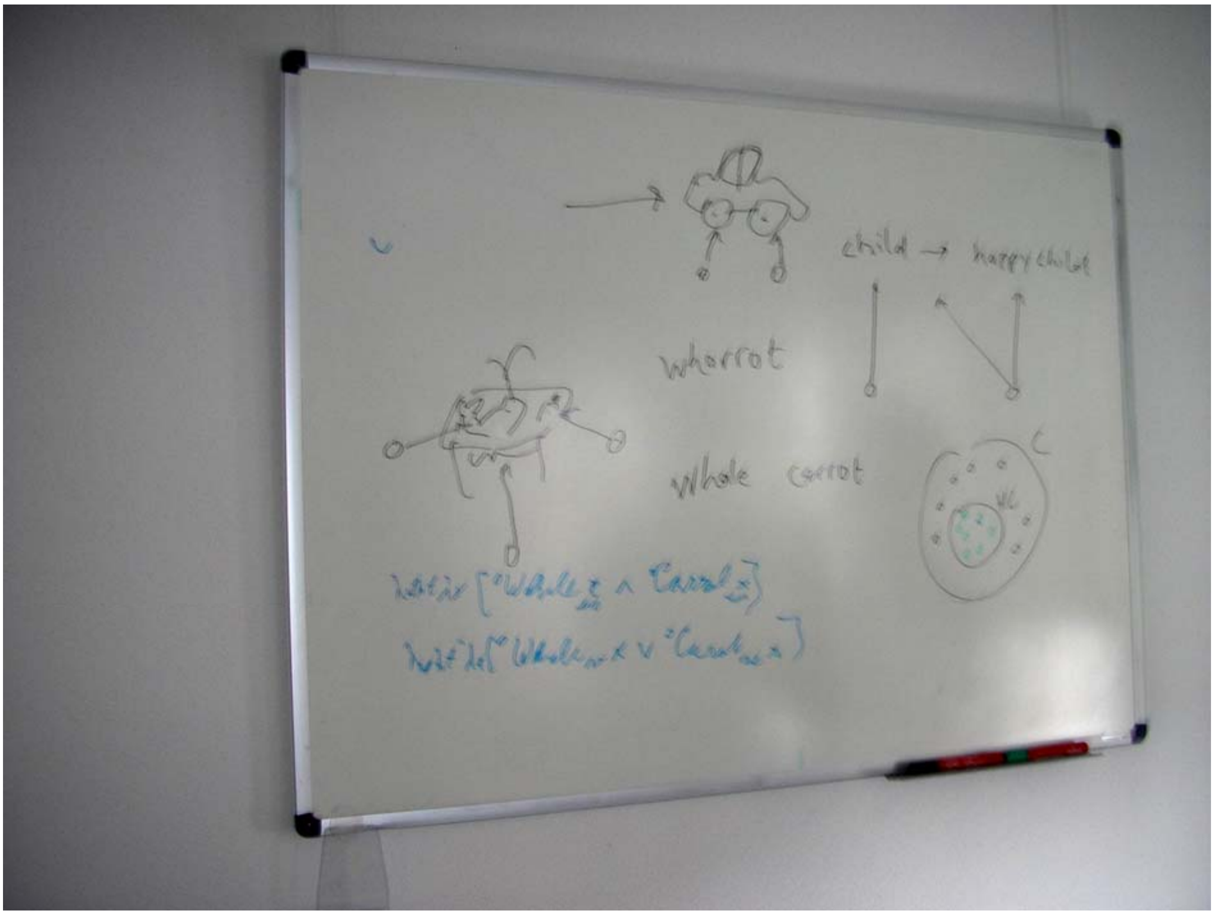
$$\text{Wharrot}_1 = \lambda w \lambda t\, \lambda x\, [[{}^0 Whale_{wt}\, x] \wedge [{}^0 Carrot_{wt}\, x]].$$

$$\text{Wharrot}_2 = \lambda w \lambda t\, \lambda x\, [[{}^0 Whale_{wt}\, x] \vee [{}^0 Carrot_{wt}\, x]].$$

$$\text{Wharrot}_3 = \lambda w \lambda t\, \lambda x\, \exists y\, \exists z\, [[{}^0 Whale_{wt}\, y] \wedge [{}^0 Carrot_{wt}\, z] \wedge [{}^0 Part\_of_{wt}\, y\, x] \wedge [{}^0 Part\_of_{wt}\, z\, x]].$$

$$\text{Wharrot}_4 = \lambda w \lambda t\, \lambda x\, \exists y\, \exists z\, [[{}^0 Whale_{wt}\, y] \wedge [{}^0 Carrot_{wt}\, z] \wedge [{}^0 Living\_creature_{wt}\, x] \wedge [{}^0 Part\_of_{wt}\, y\, x] \wedge [{}^0 Part\_of_{wt}\, z\, x]].$$

Types: *Whale, Carrot*/$(o\iota)_{\tau\omega}$; *Part_of*/$(o\iota\iota)_{\tau\omega}$;



Obviously, wharrots$_1$ could not exist. No individual (as a whole) has the property of being a whale and a carrot. Wharrots$_2$ could, indeed do exist; there are many individuals that have the property of being a whale or a carrot. Wharrots$_3$ also exist; there are pairs of a whale and a carrot. But wharrots$_4$ could not exist, because of DNA incompatibility.

Being a part of is a property of *individuals*, not of intensions. Thus there is, in general, no inheritance, no implicative relation, between the respective properties ascribed to individual parts and a whole. There is, of course, another question, namely *which parts are essential* for an individual in order to have the *property P*? For instance, an engine is a part of a car, but an engine is not a car. But the property of having an engine is essential for the property of being a car, because something designed without an engine does not qualify as a car but at most as a toy car, which is not a car. But the property of having this or that particular screw, say, is not essential for the property of being a car.[12]

But there is a more fundamental problem that has been dealt with by Cmorej in (1988). If a composition of a physical individual is contingent and variable, *which part of such an individual is essential for the individual's identity?* Cmorej examines in (1988) the problem of variable mereological sums, and he comes to the conclusion that the assumption of a variable composition of a mereological individual leads to absurd consequences. Let us briefly summarise his results.

Cmorej presents two thought experiments which set up a sort of puzzles. The first puzzle can be called "*Did an individual have the property P, or not*?"; the second one, "*Where is the individual*?"

(a) *Did an individual have a property P, or not*? Imagine an individual X that has the property P. The property P is *penetrating*, which means that, necessarily, if X has P then all its parts have P.

Formally, P is *penetrating* iff

$$\forall w \forall t\ \forall x\ [[{}^0P_{wt}\, x] \supset \forall y\ [[{}^0Part\_of_{wt}\, y\ x] \supset [{}^0P_{wt}\, y]]].$$

Types: $P/(o\iota)_{\tau\omega}$; $Part\_of/(o\iota\iota)_{\tau\omega}$; $x, y \rightarrow \iota$.

For instance, the property of weighing less than 50 kg is penetrating. An individual cannot weigh less than 50 kg if some of its parts weigh more than 50 kg.

Now let X have P at time t_1. During the time interval $\langle t_1, t_2\rangle$, $t_1 < t_2$, X loses all its *proper* parts[13] as well as P so that at t_2 X does not have P anymore, and X also does not contain any *proper* part that *used* to have P.

Now the question is whether at t_2 we can truly ascribe to X the property of having P in the past. Cmorej uses a past-tense operator ***Pt*** that is applied to the proposition that X has P, forming the proposition that X had P in the past.

The operator $\boldsymbol{Pt}/(oo_{\tau\omega})_{\tau\omega}$ is thus a property of propositions defined as follows.

Let $p \rightarrow o_{\tau\omega}$ be a variable v-constructing a proposition. Then

$${}^0\boldsymbol{Pt} = \lambda w \lambda t\ \lambda p\ \exists t'\ [[t' < t] \wedge p_{wt'}].$$

Intuitively, the answer is in the affirmative. It is true at t_2 that X used to have P, because "what is done cannot be undone". But how are we to evaluate the truth-conditions of the proposition

[12] This problem is connected with the analysis of property modification, including *being a malfunctioning P*, and we are not going to deal with it here. For a formal analysis of malfunctioning, see Duží *et al* (2007, §5.3).

[13] A proper part of X is an individual Y such that Y is a part of X and $Y \neq X$.

$\lambda w\lambda t\ [{}^0\boldsymbol{Pt}_{wt}\ \lambda w\lambda t\ [{}^0P_{wt}\ X]]$ at t_2? When evaluating the proposition $\lambda w\lambda t\ [{}^0P_{wt}\ X]$ we certainly have to consider all the parts of X, because P is penetrating. Now Cmorej argues that, similarly, when evaluating the truth-conditions of $\lambda w\lambda t\ [{}^0\boldsymbol{Pt}_{wt}\ \lambda w\lambda t\ [{}^0P_{wt}\ X]]$ at t_2 we have to take into account the parts X consists of *at time* t_2. But there is not a trace of P in X at t_2; no proper part of X used to have P. This is peculiar, indeed. Could X have been, for instance, inside a room, or in a magnetic field, or submerged into a liquid, if there is not even a tiny proper part of X to which the respective property could have been ascribed? Hardly. Thus Cmorej comes to the conclusion that $\lambda w\lambda t\ [{}^0\boldsymbol{Pt}_{wt}\ \lambda w\lambda t\ [{}^0P_{wt}\ X]]$ is, at t_2, both true (according to the principle "what is done cannot be undone") and false, because none of its parts used to have the property P. Contradiction!

First, however, I disagree with Cmorej's argument by analogy. He argues that when evaluating whether "The world champion in 100 m run used to be a smoker" we examine the *current* world champion, not the previous ones. Of course, we have to examine the individual that *currently* and *actually* plays the role of world champion in 100 m, but we examine his/her *history*. Though the current champion might have stopped smoking we would ask whether he/she *previously smoked.* Similarly, when asking whether X used to have P we have to examine the *history* of X, i.e., the proper parts X used to consist of. We have to ask which parts X consisted of in the past, and whether some of these parts previously used to have P, namely in the interval $\langle t_1, t_2\rangle$.

This follows also from the definition of the ***Pt*** property:

$$\lambda w\lambda t\ [{}^0\boldsymbol{Pt}_{wt}\ \lambda w\lambda t\ [{}^0P_{wt}X]] = \lambda w\lambda t\ \exists t'\ [[t' < t] \wedge [{}^0P_{wt'}X]].$$

Thus evaluating the truth-conditions in a world w at time t_2 means empirically searching for the truth-value constructed by $\exists t'\ [[t' < t_2] \wedge [{}^0P_{wt'}X]]$. In other words, we have to examine the history of X *before* t_2.

But there is another, more alarming question. If no current proper part of X can serve in order to examine the history of X, how then are we to examine X? We have to abstract from all the current proper parts of X, as well as all their properties, and consider only the properties the *bare* individual X used to have.

What, then, does determine the *identity* of the *bare* individual X?

(b) *Where is the individual*? The second thought experiment establishes a similar sort of puzzle. Imagine that an individual a owns a golden fountain pen (i.e., a pen, all parts of which are golden) and an individual b has a pen that looks exactly like a's, except that it is not made of gold but of fool's gold (i.e., all its parts are made of fool's gold). Moreover, b's pen and all its parts function in exactly the same way as a's pen parts and are mutually interchangeable. At time t_1 a's pen is located at the place L_a and b's pen at the place L_b. Now during the time interval $\langle t_1, t_2\rangle$ b gradually replaces, part by part, the *proper* parts of a's pen by the *proper* parts of b's pen, so that at t_2 all the *proper* parts of a's pen are located at L_b and all the *proper* parts of b's pen are located at L_a. As a result, a's pen and b's pen look and function in the same way at t_2 as they did in t_1, except of the fact that a's pen is made of fool's gold and b's pen is made of gold.

Well, this is one side of the coin. But the other side of the coin is this. Imagine that the interval $\langle t_1, t_2\rangle$ is very short and that all the parts have been interchanged *at once*. Wouldn't we be inclined to say that b simply stole a's pen and replaced it by his worthless fool's gold pen? And even if the interchange was performed part by part, how could *all* the proper parts of a's pen be transferred from L_a to another location L_b without the whole individual being transferred?

Hence the questions arise: Where is *a*'s pen and where is *b*'s pen at t_2? Which of the pens is golden at t_2? And there are two mutually incompatible answers:

i) *a*'s pen is located at L_a and is made of fool's gold, whereas *b*'s pen is located at L_b and is made of gold; *b* did not steal *a*'s pen, *b* only made *a*'s pen much less valuable.

ii) *a*'s pen is located at L_b and is made of gold, whereas *b*'s pen is located at L_a and is made of fool's gold. *b* stole *a*'s pen, and replaced it by his worthless fool's gold pen.

Now imagine that an expert examines the two pens at t_2. In *both cases* the result of the expert's evaluation would be as follows. The pen located at L_a is made of fool's gold, because *all* its parts are made of fool's gold, whereas the pen located at L_b is golden, because *all* its parts are made of gold. Everybody, even a complete fool, would claim that the golden pen at L_b is *a*'s pen. As a result, the variant *ad* (i) seems to be impossible.

Cmorej thus arrives at the conclusion that the assumption of unrestricted variation of an individual's composition is not acceptable. In other words, given an individual *X*, the property of being a part of *X* must be essential of *X*. Hence for any individual *X* it holds that the property constructed by

$$\lambda w \lambda t\ \lambda y\ [{}^0Part\_of_{wt}\ y\ X]$$

is an *essential* property of *X*, i.e., a constant function. But at the same time this property is, intuitively, *empirical.* We cannot know *a priori* which parts *X* consists of. We have to empirically investigate it.

The consequence of this conclusion is, however, too drastic. The individual *X* consists of the same parts in each world *w* at each time *t.* It means that the composition of an individual must be constant, and each time an individual *X* loses some part and obtains a new one, a *new* individual *X'* comes into being. As a result, the universe of discourse would have to vary as well. Moreover, we could not *a priori* distinguish between individuals *X*, *X*', *X*'', *X*''', etc. This is certainly untenable. Without empirically examining particular properties, we know, at least in principle, that this particular individual is different from that one, thanks to strictly numeric individuation.

But as the above thought experiments show, if we admit a variable composition of a mereological individual, then we face a problem with the identity of individuals. To dramatize the problem, imagine that somebody is gradually stealing (proper) parts of your car. If the thief steals one molecule he has not stolen your car. If he steals a wheel, he has not stolen your car. If he steals all four wheels, he has not stolen your car. But if the thief steals *all* proper parts of your car, wouldn't you say that he had stolen your car? But if so, *which* part is then essential of your car's identity?

It seems that the only way out is the answer that *no proper* physical, material part is essential. But then an individual may lose all its proper (physical, material) parts without losing its identity, and the identity of an individual is a pure *abstraction*. Similarly, a bare individual is then a pure abstraction, and Cmorej is right in his proving that bare individuals do not exist, because existence is a property of *intensions*, namely the property of being instantiated. And we cannot specify the property of *not* having any properties. We can only *abstract* away the properties an individual has. We have to pre-theoretically presuppose that there is a fixed domain of individuals whose identity is given to us *a priori*, regardless of the question whether we are always able to determine *which* particular individual we are examining on some occasion. As said above, individuals are logically primitive relative to a base *B*. And it does not make sense to ask whether bare individuals exist. Being abstract, they do not exist in space and time, but they are here, being at our disposal *a priori.*

There is another argument in favour of the hypothesis of abstract bare individuals. In case of technical artefacts, like a car or a fountain pen, the idea of an abstract individual identity seems to be peculiar. But consider living creatures, for instance human beings. Biology teaches us that every cell of a human being is physically replaced by a new one several times during his or her life cycle. If any physical part of a human being were essential of its identity, each human being would change its identity during the life-cycle period several times over. But I am confident that I am still the numerically same individual as when I was born.

What then guarantees the identity of a human being? If biologists are right, the DNA structure is unique for each human being. But the DNA structure is an abstract *instruction*, as opposed to its *realisations* by sequences of chromosomes. Hence the identity of human beings is uniquely specified by an abstract DNA structure. But then one may ask: Doesn't the DNA structure determine at least some of the properties a human being may have? For instance, a human DNA structure cannot give rise to a dog. In other words, since the property of being human is determined by the DNA structure, and the DNA structure uniquely and essentially specifies a human being's identity, the property of being human is essential of the individual with a human genome.

The answer is No. The property of being human is not essential of an individual instantiating the property. Of course, once a creature is *realised* according to a particular DNA instruction, it is necessarily realised in accordance with the instruction. But there is no sort of *logical* necessity that this or that structure be realised. It is only a sort of a *nomic* necessity: *If* the structure is realised *then* the creature must have the properties dictated by the DNA. But the truth-value of the antecedent is logically contingent.

If we accept this conception of individuals, then the cardinality of the universe is certainly very large, and the set of individuals is most probably uncountable. Of course, physical mereological sums that we empirically examine for their properties are not bare abstract entities, and in every possible world/time there are most probably countably many of them. But these are merely the *realised* abstract patterns. Bare individuals, being pure abstract patterns, might then be divided into the actually realised and unrealised ones. The latter might be conceived as Svoboda's individuals in limbo (see Svoboda (2000)).

Acknowledgement. I am grateful to Bjorn Jespersen and Pavel Materna for their valuable comments that improved the quality of the paper. I am also grateful to Maarten Franssen for stimulating ideas that we discussed in Delft. Last but not least, I am grateful to Pavel Cmorej for the rich discussion on the problem of bare individuals in 2005.

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