Tutorial 8 – Skolem clausal form, Unification

Exercise 1: Transform the following formulas into the *Skolem clausal form*:

a) $\exists x \forall y \forall z[P(x, y, z)]$

b) $\exists x \exists y \forall z[P(x, y, z)]$

c) $\forall x \exists y \forall z[P(x, y, z)]$

d) $\exists x \forall y \exists z[P(x, y, z)]$

e) $\forall x \exists y \exists z[P(x, y, z)]$

f) $\forall x \forall y \exists z[P(x, y, z)]$

g) $\forall x \exists y \forall z \exists v[P(z, y) \wedge Q(x, v)]$

h) $\forall x \exists y \forall z \exists v[P(z, y) \supset Q(x, v)]$

i) $\forall x \exists y \forall z \exists v[P(z, y) \wedge Q(x, y)]$

j) $\forall x \exists y \forall z[(P(x, y) \supset Q(y, z)) \vee Q(x, y)]$

k) $[\forall x(P(x) \supset \exists y \forall z(P(y) \wedge Q(y, z) \wedge Q(x, z)))] \supset \exists x Q(x, a)$

Exercise 2: Unify:

a) $P(x, y)$; $P(z, g(t))$

b) $P(f(x), z, g(y, a))$; $P(y, x, g(f(a), z))$

c) $P(x, b, f(x))$; $P(a, y, f(y))$

d) $P(x, f(x, z), h(a))$; $P(y, f(y, y), w)$

e) $P(x, f(y), z)$; $P(f(u), v, f(w))$ - u, v, w are variables

Exercise 3: For the following formulas, decide whether they are logically valid using Resolution method:

a) $\exists x P(x) \vee \exists x \neg P(x)$

b) $\forall x[[\neg P(x) \vee Q(x, h(x))] \wedge \neg P(f(a))]$

Exercise 4: For the following arguments, decide whether they are valid using Resolution method.

a) No one who is claustrophobic can work as a liftboy.
All climbers are claustrophobic.

Therefore no climber can work as a liftboy.

b) All wooden tables are tables.
All wooden tables are made of wood.

Some tables are made of wood.