Tutorial 5 – Set Theory.

Exercise 1: Define, make an example and draw:

a) Union of sets A and B. $(A \cup B)$

b) Intersection of sets A and B. $(A \cap B)$

c) Difference of sets A and B. $(A \setminus B)$

d) A set A is a subset of a set B. $(A \subseteq B)$

e) A complement of a set A with respect to a set M. $\overline{A}$

f) Equality of sets A and B. $(A = B)$

g) A power set of a set A. $P(A)$ nebo 2^A

h) A is a proper subset of B. $(A \subset B)$

i) Cartesian product of A and B $(A \times B)$

j) A relation between sets A and B. $(R \subseteq A \times B)$

k) Mapping from A to B. $(A \rightarrow B)$

Exercise 2: Prove for any set A and B:

a) $A = B$, iff $A \subseteq B$ a $B \subseteq A$

b) $A \subseteq (A \cup B)$

c) $(A \cap B) \subseteq A$

d) If $A = \emptyset$ and $B = \emptyset$ then $(A \cup B) = \emptyset$ and vice versa.

e) If $A = \emptyset$ then $(A \cap B) = \emptyset$ for any set B.

f) $((A \cup B) \setminus C) \subseteq (A \cup (B \setminus C))$

g) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ de Morgann's laws

h) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ de Morgann's laws

Exercise 3: Decide, whether following statements are true:

a) $a \in \{\{a\}, \{a, \{a\}\}\}$

b) $\{a, \{a\}\} \cap P(\{a, \{a\}\}) = \emptyset$

c) $\{\emptyset\} \in \{\{\emptyset\}\}$

d) $\bigcap A_i = \emptyset$, where A_i is an alement of $P(A)$, i.e. $A_i \in 2^A$.

Exercise 4: Identify all elements of following sets:

a) $\{a, \{a\}\} \cup \{a, \{b\}, c\}$

b) $\{a, \{a\}\} \cap \{a, \{b\}, c\}$

c) $\{a, \{a\}\} \setminus \{a, \{b\}, c\}$

d) $\overline{A}$ with respect to B where $A = \{a, b, \{c\}\}, B = \{a, b, \{c\}, \{a, b\}\}$ and $A \subseteq B$

Exercise 5: Decide, whether following statements are true:

a) A set of people living in Europe or Asia and not in Ural mountains is a subset of people living in Europe or Asia.

b) All citizens of Czech republic live in Moravia, Bohemia or in Silesia.

c) Some citizens of Moravia live in Silesia.

d) All people living in Europa and Asia belong into a set of people living in Asia or Europe.

Exercise 6: Decide, whether the following relations are functions, if so which kind. We will work with sets: $A = \{a_1, a_2, a_3, a_4\}$, $B = \{b_1, b_2, b_3\}$ a $C = \{c_1, c_2, c_3\}$:

a) $R \subseteq A \times B, R = \{(a_1, b_3), (a_2, b_2), (a_1, b_1)\}$

b) $R \subseteq B \times A, R = \{(b_1, a_4), (< b_2, a_4), (b_3, a_4)\}$

c) $R \subseteq A \times C, R = \{(a_1, c_2), (< a_2, c_3), (a_3, c_1)\}$

d) $R \subseteq C \times A, R = \{(c_2, a_1), (c_3, a_2), (c_1, a_3)\}$

e) $R \subseteq B \times C, R = \{(b_1, c_3), (b_2, c_2), (b_3, c_1)\}$

f) $R \subseteq A \times B \times C, R = \{(a_1, b_1, c_1), (a_2, b_2, c_1), (a_1, b_2, c_3)\}$

g) $R = \{< x, y > \in \mathbb{N} \times \mathbb{N}, y = x^2\}$

h) $R = \{< x, y > \in \mathbb{N} \times \mathbb{N}, x = y^2\}$

i) $R = \{< x, y > \in \mathbb{Z} \times \mathbb{Z}, y = x^2\}$

j) $R = \{< x, y > \in \mathbb{Z} \times \mathbb{Z}, x = y^2\}$