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RESEARCH ARTICLE

# Specification of Agents' Activities in Past, Present and Future

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*Abstract*: The behaviour of a multi-agent system is driven by messaging. Usually, there is no central dispatcher and each autonomous agent, though resource-bounded, can make less or more rational decisions to meet its own and collective goals. To this end, however, agents must communicate with their fellow agents and account for the signals from their environment. Moreover, in the dynamic, permanently changing world, agents' behaviour, i.e. their activities, must also be dynamic. By communicating with other fellow agents and with their environment, agents should be able to learn new concepts and enrich their knowledge base. Processes and events that happened in the past may be irrelevant in the present or have a significant impact in the future, and vice versa. Therefore, the fine-grained analysis of agents' activities as well as events within or beyond the system is very important so that the system can run smoothly without falling into inconsistencies. Moreover, as the system should communicate with its environment, the analysis should be as close to natural language as possible. The goal of this paper is a proposal for such an analysis. To this end, I apply Transparent Intensional Logic (TIL) because TIL is particularly apt for a fine-grained analysis of processes and events specified in the present, past

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or future tense with reference to the time when they happened, happen or will happen.

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## 1. Introduction

A multi-agent system (MAS) is a distributed system of (more or less) intelligent agents who are *active* in their perceiving environment and acting to achieve their individual and collective goals.[1] The agents are autonomous in the sense of not being controlled by a central dispatcher; the system is driven only by messaging.[2] To obtain a needed piece of information, the agents must be able to *ask* their fellow agents. Yet, they need to put forward not only *Yes-No* questions but also, in particular, *Wh*-questions. While there is just one type of answer to a Yes-no question, the class of Wh-questions is much more abundant in types. From the logical point of view, the type of possible answer determines the type of Wh-question. In regular communication, we ask by using different pronouns in interrogative sentences, and these pronouns indicate the type of possible answer. We can integrate logical and linguistic views to classify Wh-questions into more detailed classes. For instance, by 'who', we ask for a person; by 'where', for a location or position; 'when' means asking for the time. A proposal for such a more detailed classification of Wh-questions has been introduced in (Číhalová, Duží 2022). Each specialised subtype of a Wh-question conveys specific instructions for an agent on how and where to find the corresponding answer. Detailed classification of queries thus improves agents' communication and intelligent behaviour. In particular, the specific types of Wh-questions are apt for the communication of agents concerning their *dynamic*

[1] By 'intelligent' I do not mean human intelligence in case of software agents, of course. Instead, I am talking about artificial intelligence, which is actually not an intelligence, as Roger Penrose in his 1994 book argues. Anyway, in this paper I use the term 'intelligence' for both.

[2] See, for instance Wooldrige (2009).

*activities*. The agents need to know *who* is the actor of an activity, *when* the activity starts and ends, by *which* instruments it is performed, etc.

The systems of erotetic logic are valuable, as they render many exciting features of Yes-No questions and answers.[3] However, many other essential features of questions stem from their *presuppositions*. Yet, to my best knowledge, none of the systems of erotetic logic deals with Wh-questions and presuppositions of questions in a plausible way. This is unsatisfactory, as Wh-questions are even more frequent than Yes-No questions in our everyday vernacular.[4]

To obtain a literal analysis of natural language sentences, I am going to apply Tichý's (1988) Transparent Intensional Logic (TIL) with its procedural semantics, namely, its version as introduced in (Duží, Jespersen and Materna 2010). The analysis of empirical Wh-questions transforms in the TIL formalism into λ-terms denoting procedures that produce α-intensions (functions with the domain of possible worlds ω and times τ, and values of type α) where α is not a truth-value. The sought answer should provide an object of type α, which is the value of the α-intension asked for in the actual world at the time of evaluation. Since ordinary erotetic logics do not usually deal with Wh-questions, (Duží and Fait 2021) adjusted Gentzen's system of natural deduction for TIL so that the system can answer not only Yes-No questions by keyword searching but also answer Wh-questions by inferring computable knowledge from natural-language texts.[5] The paper

[3] See, for instance, Harrah (2002) or Peliš and Majer (2011). For a system based on relevant logic that can provide axioms and rules for dealing with Yes-No questions, see, for instance (Punčochář 2020).

[4] There are a few systems dealing with Wh-questions, see, for instance, Groenendijk (2003), Haida (2008), Hamblin (1973), Essberger (online) or Kartunen (1977). Yet, none of them covers this issue in a satisfactory way. Their summary and appraisal from the point of view of application in TIL can be found in Číhalová, Duží (2022).

[5] Computable or inferable knowledge has been introduced as a golden middle way between two extremes, namely explicit and implicit knowledge. Classical epistemic systems deal with explicit and implicit knowledge. The former prevents the paradox of logical/mathematical omniscience by depriving the agents of any inferential abilities, as they know only those pieces of knowledge that are explicitly recorded in their knowledge base. On the other hand, dealing with implicit knowledge

describes a useful logical technique of deriving answers to Wh-questions based on a given knowledge base that can be both an agent's base or even natural language texts. It consists of enriching the system of natural deduction with special rules rooted in the rich semantics of a natural language. In addition, special technical rules are specified to operate *into* hyperintensional contexts; see Duží, Jespersen (2015) and Jespersen, Duží (2022).

In (Číhalová, Duží 2022) the analysis of agents' activities is briefly outlined. The goal of this paper is to propose a detailed analysis of agents' dynamic activities both from the point of view of their specifications and answering questions on such activities. The analysis takes account of time, i.e. sentences in the past, present or future tenses with reference to the time when this or that happened, is happening or will happen.

The rest of the paper is organised as follows. Section 2 summarises the basic principles of Transparent Intensional Logic (TIL). In Section 3, I briefly reproduce the conceptual-oriented classification of *Wh*-questions as of (Číhalová, Duží 2022). The main novelty of this paper is presented in Section 4; it is the analysis of agents' dynamic activities specified in past, present or future tenses together with the agents' learning new concepts by questioning and answering. Concluding remarks and proposals for further research can be found in Section 5.

## 2. Basic Principles of TIL

Pavel Tichý, the Transparent Intensional Logic (TIL) founder, was inspired by Frege's semantic triangle. Frege characterised the sense of an

presupposes that the agents would be able to derive all the logical consequences of their explicitly recorded pieces of knowledge, if only they had an infinite amount of time and resources at their disposal. Hence, implicit knowledge inevitably yields the paradox of logical/mathematical omniscience. Since both notions are not realistic in case of modelling behaviour of intelligent but resource bounded agents, we introduce the notion of inferable knowledge. The idea is simple. Having an agent with some inferential abilities and an explicit knowledge base, we compute maximal limit of knowledge they are able to infer by applying the rules of inference the agent masters. For details, see Duží, Menšík (2017).

expression as the 'mode of presentation'. Tichý defines this mode of presentation as an abstract, algorithmically structured *procedure* that produces the object denoted by the expression or, in rigorously defined cases, fails to produce a denotation if there is none.[6] This is because there are non-denoting terms that have a perfect meaning, like 'the greatest prime number' or 'the value of the cotangent function at the number π'. Mathematicians had obviously to understand the sense of these terms first, and only then could they prove that there are no such numbers. Hence, in TIL, the meaning of an expression is understood as a context-invariant *procedure* encoded by a given expression. By 'context invariant', we mean this. The procedure encoded by an unambiguous expression is one and the same (up to procedural isomorphism) independently of the context in which the expression is used.[7] If the expression is ambiguous, it is furnished with more than one procedure corresponding to its different meanings.

Tichý defined six kinds of meaning procedures and called them *constructions.* There are two kinds of *atomic* constructions that supply input objects to be operated on by molecular constructions. They are *Trivialization* and *Variable.* A Trivialisation presents an object $X$ without the mediation of any other procedures. Using the terminology of programming languages, the Trivialisation of $X$, denoted by '$^0X$', is just a *pointer* or *reference* to $X$. Trivialization can present an object of any type, even another construction $C$. Hence, if $C$ is a construction, $^0C$ is said to *present* the construction $C$, whereby $C$ occurs *hyperintensionally*, i.e. in the *non-executed* mode. Variables produce objects dependently on valuations; they are said to *v-construct.* The execution of a Trivialisation or a variable never fails to produce an object. However, since TIL is a logic of partial functions, the execution of some of the molecular constructions can fail to present an object of the type

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6 See Tichý (1988). A similar philosophy of meaning as a 'generalized algorithm' can be found in (Moschovakis 2006); this conception has been further developed by Loukanova (2009). TIL procedural viewpoint is also not far from the idea of algorithmic logic, see Li, B. (2022). 4936. https://doi.org/10.20935/AL4936.

7 For the definition of procedural isomorphism, see (Duží 2019). Briefly, there is no unique criterion for procedural isomorphism and any language, any discourse. In practice, procedures are isomorphic if their specification is identical up to α-equivalence or restricted β-equivalence.

they are typed to produce. When this happens, we say that a given construction is *v-improper*.

There are two kinds of *molecular* constructions, which correspond to *λ-abstraction* and *application* in the λ-calculi, namely *Closure* and *Composition*. λ-*Closure*, $[\lambda x_1 \ldots x_n\ X]$, is the very procedure of producing a function with the values *v*-produced by the procedure $X$, by abstracting over the values of the variables $x_1, \ldots, x_n$ to provide functional arguments. No Closure is *v*-improper for any valuation *v*, as a Closure always *v*-constructs a function (which may be, in an extreme case, a degenerate function undefined at all its arguments). *Composition*, $[X\ X_1 \ldots X_n]$, is the very procedure of applying a function $f$ produced by $X$ (if any) to the tuple argument $\langle a_1, \ldots, a_n \rangle$ (if any) produced by the procedures $X_1, \ldots, X_n$. A Composition is *v*-improper as soon as $f$ is a *partial function* not defined at its tuple argument or if one or more of its constituents $X, X_1, \ldots, X_n$ are *v*-improper.[8]

TIL being a *hyperintensional* system, each construction $C$ can occur not only in execution mode so as to produce an object (if any) when being executed but also as an object in its own right on which other (higher-order) constructions operate. The Trivialisation of $C$ causes $C$ to occur just presented as an argument, as mentioned above. Yet sometimes, we need to cancel the effect of Trivialisation and trade the mode of $C$ for execution mode. *Double Execution*, $^2C$, does just that; it executes $C$ twice over. If $C$ *v*-constructs a construction $D$ that in turn *v*-constructs an entity $E$, then $^2C$ *v*-constructs $E$. Otherwise, $^2C$ is *v*-improper. Hence, for any construction $C$, this law is valid: $^{20}C = C$.

**Definition 1** (*construction*)

(i) *Variables* $x, y, \ldots$ are *constructions* that construct objects (i.e., elements of their respective ranges) dependently on a valuation function *v*; they *v*-construct.

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8 In the rest of this section, I draw on the standard exposition of the fundamentals of TIL, as presented in other papers (for instance in Jespersen, Duží (2022) or Duží, Fait (2021)), with just a few minor adjustments. True, since TIL has become a well-known system, this exposition could have been more condensed; yet, in the effort of making everything comprehensive and convenient for a reader, I leave this part in full details.

(ii) Where $X$ is an object whatsoever (even a *construction*), ${}^0X$ is the *construction Trivialisation* that constructs $X$ without any change.

(iii) Let $X$, $Y_1$, ..., $Y_n$ be arbitrary *constructions*. Then the *Composition* $[X\ Y_1 ... Y_n]$ is the following *construction*. For any $v$, the Composition $[X\ Y_1 ... Y_n]$ is *v-improper* if one or more of $X$, $Y_1$, ..., $Y_n$ are $v$-improper, or if $X$ does not $v$-construct a function that is defined at the $n$-tuple of objects $v$-constructed by $Y_1$, ..., $Y_n$. If $X$ does $v$-construct a $v$-proper function, then $[X\ Y_1 ... Y_n]$ $v$-constructs the value of this function at the $n$-tuple.

(iv) (λ-) *Closure* $[\lambda x_1 ... x_m\ Y]$ is the following *construction*. Let $x_1$, $x_2$, ..., $x_m$ be pair-wise distinct variables and $Y$ a *construction*. Then $[\lambda x_1 ... x_m\ Y]$ $v$-constructs the function $f$ that takes any members $B_1$, ..., $B_m$ of the respective ranges of the variables $x_1$, ..., $x_m$ into the object (if any) that is $v(B_1/x_1,...,B_m/x_m)$-constructed by $Y$, where $v(B_1/x_1,...,B_m/x_m)$ is like $v$ except for assigning $B_1$ to $x_1$, ..., $B_m$ to $x_m$.

(v) Where $X$ is an object whatsoever, ${}^1X$ is the *construction Single Execution* that $v$-constructs what $X$ $v$-constructs. Thus, if $X$ is a $v$-improper construction or not a construction as all, ${}^1X$ is $v$-improper.

(vi) Where $X$ is an object whatsoever, ${}^2X$ is the *construction Double Execution*. If $X$ is not itself a *construction*, or if $X$ does not $v$-construct a *construction*, or if $X$ $v$-constructs a $v$-improper *construction*, then ${}^2X$ is $v$-improper. Otherwise ${}^2X$ $v$-constructs what is $v$-constructed by the *construction* $v$-constructed by $X$.

(vii) Nothing is a *construction*, unless it so follows from (i) through (vi).

With constructions of constructions, constructions of functions, functions, and functional values in TIL stratified ontology, we need to keep track of the traffic between multiple logical strata. The *ramified type hierarchy* discharges that task. The type of first-order objects includes all objects that are not constructions. Therefore, it includes not only the standard objects of individuals and truth values but also sets, functional mappings and functions defined on possible worlds (i.e., the *intensions* germane to possible-world semantics, PWS intensions). The type of second-order objects includes constructions of first-order objects and functions with such constructions in their domain or range. The type of third-order objects includes constructions of first- or second-order objects and functions with such constructions in their domain or range; and so on ad infinitum.

**Definition 2** (*ramified hierarchy of types*)**.** Let $B$ be a *base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:

$\mathbf{T_1}$ (*types of order 1*).

i) Every member of $B$ is an elementary *type of order 1 over B.*

ii) Let $\alpha$, $\beta_1$, ..., $\beta_m$ ($m > 0$) be types of order 1 over $B$. Then the collection ($\alpha\ \beta_1 \ldots \beta_m$) of all $m$-ary partial mappings from $\beta_1 \times \ldots \times \beta_m$ into $\alpha$ is a functional *type of* order 1 *over B.*

iii) Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii).

$\mathbf{C}_{\boldsymbol{n}}$ (*constructions of order n*)

i) Let $x$ be a variable ranging over a type of order $n$. Then $x$ is a *construction of order n over B.*

ii) Let $X$ be a member of a type of order $n$. Then ${}^0X$, ${}^1X$, ${}^2X$ are *constructions of order n over B.*

iii) Let $X$, $X_1$, ..., $X_m$ ($m > 0$) be constructions of order $n$ over $B$. Then $[X\ X_1 \ldots X_m]$ is a *construction of order n over B.*

iv) Let $x_1$, ..., $x_m$, $X$ ($m > 0$) be constructions of order $n$ over $B$. Then $[\lambda x_1 \ldots x_m\ X]$ is a *construction of order n over B.*

v) Nothing is a *construction of order n over B* unless it so follows from $\mathbf{C}_{\boldsymbol{n}}$ (i)-(iv).

$\mathbf{T}_{\boldsymbol{n+1}}$ (*types of order n* + 1)

Let $*_n$ be the collection of all constructions of order $n$ over $B$. Then

i) $*_n$ and every type of order $n$ are *types of order n* + 1.

ii) If $m > 0$ and $\alpha$, $\beta_1$, ..., $\beta_m$ are types of order $n + 1$ over $B$, then ($\alpha$, $\beta_1$, ..., $\beta_m$) (see $T_1$ ii)) is a *type of order n* + 1 *over B.*

iii) Nothing is a *type of order n + 1 over B* unless it so follows from (i) and (ii).

For the purposes of natural-language analysis, we are usually assuming the following base of ground types:

$o$: the set of truth-values {**T**, **F**};

$\iota$: the set of individuals (the universe of discourse);

$\tau$: the set of real numbers (doubling as times);

$\omega$: the set of logically possible worlds (the logical space).

We assume that the universe of discourse $\iota$ is multi-valued and consists of at least two elements, though here I leave aside the cardinality of this basic type.

Empirical expressions denote *empirical conditions*, which may or may not be satisfied at the world/time pair selected as points of evaluation. These empirical conditions are modelled as (PWS-)intensions. *Intensions* are entities of type $(\beta\omega)$: mappings from possible worlds to an arbitrary type $\beta$. The type $\beta$ is frequently the type of the *chronology* of $\alpha$-objects, i.e., a mapping of type $(\alpha\tau)$. Thus $\alpha$-intensions are mostly functions of type $((\alpha\tau)\omega)$, abbreviated as '$\alpha_{\tau\omega}$'.[9] *Extensional entities* are entities of a type $\alpha$ where $\alpha \neq (\beta\omega)$ for any type $\beta$. Where the variable $w$ ranges over $\beta$ and $t$ over $\tau$, the following outline of a Closure essentially characterises the logical syntax of empirical language: $\lambda w \lambda t$ [...*w*....*t*...].

*Examples* of frequently used $\alpha$-intensions are: *propositions* of type $o_{\tau\omega}$, *properties of individuals* of type $(o\iota)_{\tau\omega}$, *binary relations-in-intension between individuals* of type $(o\iota\iota)_{\tau\omega}$, *offices* of type $\iota_{\tau\omega}$ and hyperintensional *attitudes* of type$(o\iota*_n)_{\tau\omega}$. Logical objects like *truth functions* and *quantifiers* are extensional: $\wedge$, $\vee$, $\supset$ are of type $(ooo)$, and $\neg$ of type $(oo)$.

[9] We define (PWS-)intensions as functions with the domain of possible worlds. True, most frequently, time plays the role of the second modal parameter, though not always. For instance, assuming that physical laws of nature are nomically but not analytically necessary, as physics is an empirical science, we model these intensions by construction of this form: $\lambda w\ \forall t$ [...] $\rightarrow o_{\omega}$.

The *quantifiers* $\forall^{\alpha}$, $\exists^{\alpha}$ are type-theoretically polymorphic total functions of type $(o(o\alpha))$, for an arbitrary type $\alpha$, defined as follows. The *universal quantifier* ($\forall^{\alpha}$) is a function that associates a class $A$ of $\alpha$-elements with **T** if $A$ contains all elements of the type $\alpha$, otherwise with **F**. The *existential quantifier* ($\exists^{\alpha}$) is a function that associates a class $A$ of $\alpha$-elements with **T** if $A$ is a non-empty class, otherwise with **F**.

*Notational conventions.* Below all type indications will be provided outside the formulae in order not to clutter the notation. Moreover, the outermost brackets of Closures will be omitted whenever no confusion can arise. Furthermore, '$X/\alpha$' means that an object $X$ is (a member) of type $\alpha$. '$X \rightarrow \alpha$' means that $X$ is typed to *v*-construct an object (if any) of type $\alpha$. Throughout, it holds that the variables $w \rightarrow \omega$ and $t \rightarrow \tau$. If $C \rightarrow \alpha_{\tau\omega}$ then the frequently used Composition $[[C\ w]\ t]$, which is the *extensionalization* of the $\alpha$-intension *v*-constructed by $C$, is encoded as '$C_{wt}$'. When no confusion arises, I am going to use the standard infix notation without Trivialisation for the application of logical objects like truth functions and quantifiers. Hence, instead of '$[^0\forall\lambda x\ B]$', '$[^0\exists\lambda x\ B]$', I will often write '$\forall x\ B$', '$\exists x\ B$' for any $B \rightarrow o$ to make quantified formulas easier to read.

The *general semantic schema* involving the *meaning* (i.e., a construction) of an expression $E$, *denotation* (i.e., the object, if any, denoted by $E$) and *reference* (i.e., the value of an intension, if the denotation is an intension, in the actual world at the present time) is depicted by Fig. 1.

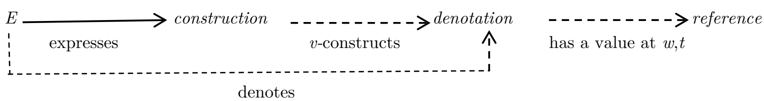


Fig. 1. TIL General semantic schema

Once the meaning construction of a term or expression has been given, it can be derived what the construction produces (if anything), i.e. what the denotation of $E$ is. Provided the denotation is not a trivial (i.e., constant) intension or a mathematical function, the reference cannot be logically derived; instead, it must be established by extra-logical and extra-semantic means (i.e., empirical inquiry or mathematical calculation) what the reference, if any, is.

As mentioned above, TIL is a logic of partial *functions*. Therefore, sets and relations are modelled by their characteristic functions. For instance, (οτ) is the type of a set of numbers, while (οττ) is the type of a binary relation-in-extension between numbers. That an element *v*-constructed by $a \rightarrow \iota$ belongs to a set $M \rightarrow (o\iota)$, which in set-theoretical notation is written as '$a \in M$', is in TIL recorded as an application of the function *M* to *a*: [*M a*]. For instance, having the set of prime numbers *Prime*/(οτ), the sentence "2 is a prime number" is furnished with this simple construction as its meaning: $[{}^0Prime\ {}^02]$.

Note that any non-procedural entities must be supplied to molecular constructions by Trivialization (or a variable, as the case may be). The reason is this. Parts or constituents of procedures can be only their (sub)procedures. No non-procedural abstract or concrete object can be a constituent part of a procedure. The objects on which procedures operate are beyond them. Thus, while *John* is an individual that cannot be executed and thus cannot be a part of a procedure, ${}^0John$ is a procedure, albeit trivial.[10]

Properties of individuals are intensions, objects of type $(o\iota)_{\tau\omega}$. In order to apply a property to an individual, a functional application is used. However, properties are not type-theoretically proper entities to be directly applied to an individual. They have to be extensionalized first. For instance, the sentence

"John is a surgeon"

ascribes the property of being a surgeon to John. As with any other non-procedural objects to be operated on, the individual John, as well as the property of being a surgeon, are supplied by their Trivialisation, ${}^0John$, ${}^0Surgeon$. Since the property is an intension of type (((οι)τ)ω), or $(o\iota)_{\tau\omega}$ for short, the property must be applied to a possible world (type ω) first and then to time (type τ). To this end, we have variables $w \rightarrow \omega$ and $t \rightarrow \tau$; thus, we get $[[{}^0Surgeon\ w]\ t]$, or ${}^0Surgeon_{wt}$, for short. In this way, we obtain the population of surgeons in the world *w* and time *t*, in which we are going

[10] In this paper, I do not deal with the semantics of proper names. Whenever used here, a proper name simply stands for a label of an individual. For the viewpoint on the TIL semantics of proper names, see Jespersen & Zouhar (1999) or Zouhar (2000).

to evaluate the truth value of the sentence. That John belongs to this population is expressed simply by the application of this population to John: $[{}^0Surgeon_{wt}\ {}^0John] \rightarrow$ ο. Finally, we abstract over the values of the variables $w$ and $t$ to obtain the proposition that John is a surgeon.

$$\lambda w \lambda t\ [{}^0Surgeon_{wt}\ {}^0John] \rightarrow o_{\tau\omega}$$

So much for the basic technicalities of TIL.

Other ingredients that I need to illustrate the communication of agents, their reasoning and learning by messaging are the notions of requisite and refinement. (Duží et al. 2010, Ch. 4) introduces a logic of intensions that has been developed into an *intensional essentialism* which spells out how some intensions supervene on other intensions.[11] The key notion is that of *requisite.* Intuitively, a requisite of an intension $A$ is a further intension $B$ that must, as a matter of analytic necessity, be possessed by any entity that happens to be in the extension of $A$. For instance, the property of being unmarried is a requisite for having the initial property of being a bachelor; if an individual $a$ happens to be a bachelor, then it must be unmarried. Formally, a requisite is a relation-in-extension between intensions of any type, though typically between individual properties or offices. For the sake of simplicity, here I define the relation of requisite between individual properties of type $(o\iota)_{\tau\omega}$. Since TIL is a logic of partial function, to deal with partiality properly, we need to apply the property $True/(oo_{\tau\omega})_{\tau\omega}$ of propositions. The reason is this. Propositions can have *truth-value gaps* in some worlds and times; in such a case, the extensionalisation of the proposition $P$, i.e. $P_{wt}$, fails to produce a truth-value, the Composition is $v$-improper. Partiality, as we all know very well, brings about technical complications. To deal with them, we define three properties of propositions *True*, *False* and *Undefined*, all of type $(oo_{\tau\omega})_{\tau\omega}$, as follows ($P \rightarrow o_{\tau\omega}$):

$[{}^0True_{wt}\ P]$ $v$-constructs **T** if $P_{wt}$ $v$-constructs **T**, otherwise **F**;

$[{}^0False_{wt}\ P]$ $v$-constructs **T** if $\neg P_{wt}$ $v$-constructs **T**, otherwise **F**;

$[{}^0Undefined_{wt}\ P] = \neg[{}^0True_{wt}\ P] \wedge \neg[{}^0False_{wt}\ P]$.

[11] Intensional essentialism obtains between intensions, unlike individual anti-essentialism that concerns bare individuals.

**Definition 3** (*requisite*). Let *f*, $g \rightarrow (o\iota)_{\tau\omega}$ be constructions *v*-constructing properties; $True/(oo_{\tau\omega})_{\tau\omega}$ the property of a proposition of being true in a given world *w* and time *t*; $x \rightarrow \iota$; $Req/(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$. Then the *property v-constructed* by *f* is *a requisite of the property v*-constructed by *g* iff

$$[{}^0Req\ f\ g] = \forall w \forall t\ \forall x\ [[{}^0True_{wt}\ \lambda w \lambda t\ [g_{wt}\ x]] \supset [{}^0True_{wt}\ \lambda w \lambda t\ [f_{wt}\ x]]].$$

*Remark.* This definition applies the property *True* to a proposition because the relation obtains necessarily.[12] If we carelessly defined the relation by way of $\forall w \forall t\ \forall x\ [[g_{wt}\ x] \supset [f_{wt}\ x]]$, the result would be a falsehood. The reason is that, at those worlds and times at which the Composition $[g_{wt}\ x]$ or $[f_{wt}\ x]$ is *v-improper*, the universal quantifiers would return the truth value F.

The property of propositions *True* is also applied in the definition of the difference between a *presupposition* and *mere entailment.*

**Definition 4** (presupposition vs mere entailment)
Let *P*, *Q* be constructions of propositions. Then

*Q is entailed by P* iff

$$\forall w \forall t\ [[{}^0True_{wt}\ P] \supset [{}^0True_{wt}\ Q]];$$

*Q is a presupposition of P* iff

$$\forall w \forall t\ [[[{}^0True_{wt}\ P] \vee [{}^0False_{wt}\ P]] \supset [{}^0True_{wt}\ Q]].$$

As a *corollary*, we have:

*Q is a presupposition of P* iff

$$\forall w \forall t\ [\neg[{}^0True_{wt}\ Q] \supset [{}^0Undefined_{wt}\ P]].$$

If a presupposition of a proposition *P* is not true, then *P* has no truth value.

[12] Indeed, the requisite relation obtains by analytical necessity, in all possible worlds. In artificial intelligence, a weaker condition is sometimes applied; then it means ‘typically’. These typical properties related to an initial property are usually defined by means of defaults; for instance, the typical property of a bird is flying, unless it is a penguin or ostrich. For details, see Duží, Číhalová and Menšík (2011).

The relation of *refinement* obtains between *concepts*, i.e. closed constructions in their normal form.[13] Usually, we need to refine an atomic concept, i.e. Trivialisation of an entity. For instance, the atomic concept of the property of being a bachelor is $^0Bachelor$. Its refinement is an *ontological definition* of this property, where ontological definition is a molecular construction of the same property, like, for example

$$\lambda w \lambda t\ \lambda x\ [[^0Unmarried\ ^0Man]_{wt}\ x].$$

**Definition 5** (*refinement of a construction*) Let $C_1$, $C_2$, $C_3$ be constructions. Let $^0X$ be an atomic concept of $X$, and let $^0X$ occur as a constituent of $C_1$. If $C_2$ differs from $C_1$ only by containing in lieu of $^0X$ an ontological definition of $X$, then $C_2$ is a *refinement of* $C_1$. If $C_3$ is a refinement of $C_2$ and $C_2$ is a refinement of $C_1$, then $C_3$ is a *refinement of* $C_1$.

For the needs of agents' communication, we introduce the function-in-intension $Refine/(*_n *_m)_{\tau\omega}$ assigning to a construction/concept its refinement; $[^0Refine_{wt}\ ^0C] = {}^0D$ means that the construction $D$ is a refinement of the construction $C$. Note that here we make use of the *hyperintensional* features of TIL. Constructions $C$ and $D$ do not occur in the execution mode; their products are irrelevant here. Rather, they are *presented* as arguments of the function *Refine*. Therefore, they must be supplied by Trivialization.

## 3. Different kinds of Wh-questions

*Empirical* questions denote non-constant α-*intensions* of type $\alpha_{\tau\omega}$ that is functions with the domain of possible worlds. The direct answer to such a question is the value of type α of this intension in the actual world $w$ and

[13] Concept and the normal form of a construction are rigorously defined in (Duží et al. 2010, §2.2.1). Briefly, the normal form of a construction $C$ is the representant of the class of constructions that are procedurally isomorphic with $C$. It is defined as the alphabetically first, non-η-reducible construction.

time $t$ of evaluation.[14] Hence, the type of a possible direct answer dictates the type of content of an empirical question.

Empirical Yes-No questions denote *propositions* of type $o_{\tau\omega}$, where $o$ is the type of truth values.[15] The inquirer wants to know the truth-value of the proposition in question in the world $w$ and time $t$ of evaluation. For instance, the answer to the question "Is John a surgeon?" is Yes/No according as the proposition that John is a surgeon is true in $w$ and $t$. On the other hand, the variety of possible answers to Wh-questions is much greater depending on the type $\alpha$ of an $\alpha$-intension the value of which is asked for. For instance, one can ask for the value of an *individual office* (or *role*) of type $\iota_{\tau\omega}$, like "Which is the highest mountain in Slovakia?", "Who is the mayor of the city of Dunedin?", "Who is the No.1 player in ATP tennis singles"? A possible direct answer to such a question is a unique individual (an object of the type $\iota$) who happens to play a given role. For instance, the meaning of the question "Who is the mayor of the city of Dunedin?" comes down to this construction.

$$\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda who\ [who = [{}^0 Mayer\text{-}of_{wt}\ {}^0 Dunedin]]] \rightarrow \iota_{\tau\omega}$$

Types. $\mathrm{I}/(\iota(o\iota))$: the singularizer, i.e. the function that associates a set $S$ of individuals with the only member of $S$ provided $S$ is a singleton, and otherwise (if $S$ is an empty or a multi-valued set) the function I is undefined; $who \rightarrow \iota$: the variable ranging over individuals such that the individual plays the role of the Mayor of Dunedin in the world $w$ and time $t$ of evaluation (the direct answer should be provided by the valuation of this variable); $Mayer\text{-}of/(\iota\iota)_{\tau\omega}$: an attribute, i.e. an empirical function that associates a given individual with another individual (in this case that one who is a Mayer of something); $Dunedin/\iota$.

[14] (Duží, Číhalová 2015) distinguishes between *direct* and *complete* answer to an empirical question. *Direct* answer is an object $X$ of type $\alpha$ that is the value (in the world and time of evaluation) of the $\alpha$-intension asked for, while *complete answer* is the proposition that the value of the asked intension is the object $X$. The authors deal with presuppositions of questions. Their main thesis is this. If a presupposition of a given question is not true, then there is no direct answer. Instead, a plausible complete answer is the negated presupposition.

[15] For details on TIL analysis of questions and answers see (Duží et al. 2010, §3.6.).

Note that the question transforms into a construction of an individual office, as it should be. The agent would like to know the value of this office.

Another frequent type of intensions is the *property of individuals*, an object of type $(o\iota)_{\tau\omega}$. For instance, the direct answer to the question "Which are the private hospitals located in Lowestoft?" should convey a set (of type $(o\iota)$) of individuals. There are two kinds of possible direct answers. An *exhaustive answer* conveys a complete list of individuals with the property of being a private hospital in Lowestoft, while an incomplete answer provides just some of them. Anyway, in both cases, the answer should be *conclusive*; it means that the individuals belonging to this list should be referred to directly. An indirect description of an individual would not be satisfactory.[16] For instance, the answer "They are the private hospitals located in the most eastern city of England" is not conclusive. The agent would have to go on asking, "Which is the most eastern city of England?" and "Which are the private hospitals in the most eastern city of England?" and so on.

Thus, the exhaustive answer to the question would be, for instance, the set: {Carlton Court, Airey Close, Beccles Hospital Inpatients, East Point Consulting Rooms, Andaman Surgery, James Paget Hospital, East Coast Community, The Veterinary Surgery, Crest View Medical Centre}.
The analysis of the question that constructs a property of individuals (that are asked for) is this.

$$\lambda w \lambda t\, [\lambda x\, [[[{}^0\mathit{Private}\ {}^0\mathit{Hospital}]_{wt}\, x] \wedge [{}^0\mathit{Located\text{-}in}_{wt}\ x\ {}^0\mathit{Lowestoft}]]] \rightarrow (o\iota)_{\tau\omega}$$

Types. $x \rightarrow \iota$: the variable ranging over individuals; $\mathit{Private}/((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$: property modifier: an analytic function that assigns to a property another (modified) property;[17] $\mathit{Hospital}/(o\iota)_{\tau\omega}$; $\mathit{Located\text{-}in}/(o\iota\iota)_{\tau\omega}$; $\mathit{Lowestoft}/\iota$.

One can also ask for the value of an attribute at an argument like the salary of somebody. The possible answer to the question "What is John's salary?" is a number, and the question denotes a magnitude of type $\tau_{\tau\omega}$.

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16 This problem has been dealt with in Duží (2022).

17 The analyses of property modifiers has been introduced in Jespersen, Carrara, Duží (2017) or in Duží (2017).

### *3.1 Classification of Wh-questions*

Číhalová & Duží (2022) introduce the classification of Wh-questions based on the type of a possible answer. They show that for our purpose, the linguistic classifications are too coarse-grained and non-plausibly oriented. For the needs of a multi-agent system, we classify questions not only from the linguistic point of view but also from the logical point of view, with respect to a domain of interest and the structure of the agent's knowledge base. The authors distinguish between *static entities,* like necessary relations between properties of individuals and *dynamic entities,* like activities which form processes. Active actions and passive events are *activities*. Each activity can involve other objects that are called their *participants*.

The specification of *activities* is based on the linguistic theory of *verb valency frames.*[18] From the logical point of view, we deal with the verb phrases as denoting a *function* that is applied to its arguments. The number of arguments is controlled by the content verb *valency*. There are several types of valency. An *impersonal* (avalent) verb has no subject or a dummy subject. "It rains." is a typical example. Here the grammatic subject 'it' is just a dummy subject because it does not refer to any concrete object.[19] An *intransitive* (monovalent) verb has just one argument, the *subject* S; "John

[18] For the linguistic theory of verb valency frames, see Horák (1998) or Rambousek, Hlaváčková (2011). Číhalová (2016) proposed ontology of events based on the theory of verb valency frames. This theory is not unlike Chomsky's θ-theory, which is concerned with the distribution and assignment of thematic roles to arguments. The theta criterion describes the specific match between arguments and thematic roles in the logical form of a sentence. (I am grateful to the anonymous reviewer for drawing my attention to this theory.) Yet, since our research is a part of a broader project on linguistic and logical natural language analysis and processing, and since in this project we cooperate with the centre for computational linguistics in Masaryk University of Brno, we vote for the theory of verb valency frames. This theory is supported by the centre, where the lexicon of verb valencies (VerbaLex) has been developed.

[19] Lots of languages, including Romance and Slavonic ones, drop the dummy subject ('it', 'es', ...) altogether, and make sentences just with a verb in third person singular.

(S) is singing." A *transitive* (divalent) verb has two arguments, an agent (A) and a patient (P), as in "John (A) kicked the ball (P)." A *ditransitive verb* has three arguments, an agent and two patients, for instance, in "John (A) passed the ball (P) to Tom (P)." There are also a few verbs with more than three arguments (polyvalent, like tritransitive); yet they mostly arise by valency increasing, where causatives or applicatives are typical valency increasing devices.[20]

Verb valency frames determine the obligatory and facultative arguments, i.e. thematic roles of a given verb, together with their types. Facultative arguments can be missing, of course. For instance, the verb 'buy' can occur in several sentences with a different number of arguments like "Tom bought a book", "Tom bought a book in Paris", "On Friday, Tom bought a book", "Tom bought a book for Jane in Paris", etc. In our analysis, we have to take these varieties into account. Linguists have created many classifications based on verb valency frames, for instance, VALLEX or VerbaLex.[21]

John Sowa (2000) proposed a specification tool for knowledge representation, where he adopted a linguistic approach to verbs. He developed the system of conceptual graphs in which Peirce's logic is combined with the semantic networks known from artificial intelligence. For the valency participants, Sowa uses the term '*thematic roles*' or '*case relations.*' His summary of all the thematic roles can be found in (Sowa 2000, pp. 506-510) or in the web source *Thematic roles.* Sowa distinguishes several types of thematic roles, for instance, Agent, Beneficiary, Destination, Duration, Effector, Experiencer, Instrument, Location, Matter, Patient and so on.[22] Thematic role or the type of a participant expresses the role that a noun phrase plays for the activity described by a governing verb. From the viewpoint of logic, it is the relation between two entities where one is an activity (expressed by the verb), and the other is an attribute (expressed mostly by a noun, adverb, number or adjective).

The number and the categories of participants depend on the respective domain of interest and the functions of the system of agents. In this paper,

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[20] For details, see Dixon (2000).

[21] See, for instance Lopatková et al. (2006) and Hlaváčková, Horák (2006).

[22] For details, see Sowa (2000, 508-510).

I will use the following frequent kinds of attributes that can be assigned to an activity:

Pat – object affected by the activity

Ben – beneficient (somebody who has benefited from the activity)

Man – the manner of the activity execution (measure, speed etc.)

Inst – instrument

Time – when

Loc – the place of activity

Dir1 – the direction of activity – *from where*

Dir2 – the direction of activity – *which way*

Dir3 – the direction of activity – *where to*

Wh-questions concern the participants of activities; we ask for their values in a world and time of evaluation. Hence, we can distinguish questions about the process itself (*what* is going on?) from Wh-questions on the primary agent and other participants of a given activity. For instance, assume we have the sentence "John (the agent) is going (the activity) to London (Dir3) by car (Inst) in an average speed of 50 miles per hour (Man)." Then we can ask, "What is John doing?", "Who is going to London?", "How quickly does John go to London?" etc.

### *3.2 Hyperintensional questions about concepts*

A particular category of questions concerns hyperintensional questions about a given *concept.* The agents should be able to *learn* from experience through mutual communication with their fellow agents. In such a communication, it may happen that a receiving agent *b* does not 'know' a concept that is a constituent of a sender's message. By 'knowing a concept' *C,* we mean having the concept *C* in one's ontology. In such a situation, the receiving agent *b* can ask for an explication or a definition of the unknown concept. When asking for the explication of concept *C* the agent does not talk about the object produced by *C.* Rather, the concept, i.e. the closed construction *C* itself, is a subject matter that is asked for. Such a context where the construction *C* is just *presented* as an argument rather than *executed* to produce an object is *hyperintensional.* In (Duží & Vojtáš 2008), a special kind of question is introduced, namely a question with the performative *Unrecognized*, the argument of which is an unknown concept *C.* The answer is then of type *Refine*, where the message provides a concept *C'*, which refines the unknown concept *C.*

*Refinement* has been rigorously defined above (Def.5). Briefly, by refining an atomic concept of an object *O*, we mean discovering a molecular concept that produces the same object *O*. In mathematics, refining usually concerns definitions like "a *group* is a set *G* equipped with a binary operation that combines any two elements of *G* to form another element of *G* in such a way that group axioms are satisfied, namely associativity, the existence of the neutral element in *G* and invertibility." Here the atomic concept to be refined is that of a 'group'. The molecular concept refining 'group' is encoded by the definiens, namely 'a set *G* equipped with a binary operation that combines any two elements of *G* to form another element of *G* in such a way that group axioms are satisfied, namely associativity, the existence of the neutral element in *G* and invertibility'. In the case of *empirical* concepts, it is more plausible to speak about *explication.* The reason is this. To say that a molecular concept *C* is a refinement of an atomic empirical concept *D* is risky. It would be a refinement only if the molecular concept *C* were *analytically equivalent* to the original concept *D*, which means that both are the concepts of the same object $O/\alpha_{\tau\omega}$. However, in the most interesting cases of *empirical* concepts of PWS-*intensions* we use a Carnapian *explication* rather than a definition proper. Then equivalence is undoubtedly not guaranteed, for one can hardly check the identity of the intensions produced by the two concepts. Rather, a new molecular concept *C* (explicatum) should define an intensional object *O* that is as close as possible to the object referred to by an inexact (prescientific) concept *D* (explicandum).

In *Meaning and Necessity* (1947), Carnap characterises explication as follows:

> The task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather of replacing it by a newly constructed,

> more exact concept, belongs among the most important tasks of logical analysis and logical construction. We call this the task of explicating, or of giving an *explication* for, the earlier concept [...] (Carnap 1947, pp. 7-8)

Keeping this difference in mind, I use the term 'refinement' for both cases, including the explication of empirical concepts. In most cases of explicating the concept unknown to an agent, this simplification is harmless.

## 4. Agents' dynamic activities

The basic idea of the analysis is due to (Tichý 1980). Its adjustment and simplification are introduced in (Duží 2010). Tichý draws a distinction between *episodic* and *attributive* verbs. Attributive verbs ascribe properties to individuals, and their structure is usually a copula followed by an adjective or noun; for instance, 'is happy', 'is red', 'looks speedy', 'is a student' are attributive verbs. On the other hand, episodic verbs express actions performed by objects. For instance, if John is getting up, it would be insufficient to analyse this activity by assigning the property of getting up to John. Rather, John is *doing* the activity of getting up. For example, the sentence "*John is driving from Brussels to Paris at the average speed of 90 km/h*" should be analysed as describing a time-consuming *process* consisting of a series of *actions* and *events.* In (Číhalová, Štěpán 2014), the basic idea of specifying event ontology by means of verb valency frames was introduced, and (Číhalová, 2016) proposed its further adjustment. It consists, in particular, in refining the type of action executed within a given process. For instance, the specification of the process *Charles is driving from Prague to München by train at the speed of 90 km/h* is determined by the sense of the verb 'to drive' together with its arguments (*who* is driving – the actor, *when* is (s)he driving, *from where*, *to where*, by *what kind of a vehicle*, in *which speed,* etc.).

### *4.1 Agents' activities in the present*

From the logical point of view, an episodic verb denotes a relation-in-intension *Do* between an individual of type ι (the actor) and an activity.

Using a general placeholder α for the type of activity, *Do* thus obtains the type $(o\iota\alpha)_{\tau\omega}$.[23]

As mentioned above, each activity has several *participants* (i.e. assignments of an attribute to the activity), and the valency of the verb determines the compulsory participants and the maximal number of facultative participants. The attributes can be of various kinds like individuals, properties, quantities, etc. Typical kinds of attributes have been specified above. They are *Pat* (object affected by the activity), *Ben* (who has a benefit from the activity), *Manner* (manner of the activity execution), *Inst* (instrument), *Time* (when), *Time1* (time when the activity started), *Time2* (time when the activity ended), *Loc* (location of the activity), *Dir1* (direction of event – from where), *Dir2* (direction of event – where through), *Dir3* (direction of event – where to). If needed, other kinds of attributes can be specified. For the purpose of the system implementation, we only must keep the selected keywords fixed.

The type of assigning an attribute to an activity is the relation in intension between an object of type β and the activity (type α); where β can be a property of individuals like being a train, or a number of type τ (time), individual ι (like John, Prague, Brussels) etc., according to the kind of an attribute. Thus, we have a general type of participant $Part/(o\beta\alpha)_{\tau\omega}$. It must be a relation-*in-intension*, as one and the same activity can be performed with different instruments at different times, and so like. For instance, John can go from Prague to Brussels by train, and next time he can vote for a plane.

[23] In this paper, I often release typing and use instead placeholders like α, β, δ for entities too complicated from the typing point of view. As we all know well, typed languages and calculi are useful and easy to work with because typing prevents a user from making silly mistakes when specifying procedures. Yet, too strong typing can sometimes be restrictive. For this reason, typed functional programming languages are usually polymorphic, or type control is not too strict; in case of a typing error, the interpreter only informs the programmer and leaves the decision to them. As TIL is a typed lambda calculus, in its computational variant TIL-Script, we also aim to implement such useful features. Proposals of the polymorphic TIL system have been introduced in Duzi (1993), Pezlar (2020) and Pezlar (2022). For a benevolent type checking algorithm, see, e.g., Duží,Marie & Fait,Marie (2019).

A *general pattern* for the analysis of an activity $P \rightarrow \alpha$ with the actor $A \rightarrow \iota$ and participants $Part\text{-}i/(o\beta\alpha)_{\tau\omega}$ that assign attributes $X_i \rightarrow \beta_i$ to $P$ is this:[24]

$$\lambda w \lambda t\ [[{}^0Do_{wt}\ A\ \ P] \wedge [{}^0Part\text{-}1_{wt}\ X_1\ P] \wedge$$
$$[{}^0Part\text{-}2_{wt}\ X_2\ P] \wedge ... \wedge [{}^0Part\text{-}n_{wt}\ X_n\ P]]$$

For instance, the analysis of the sentence "*John goes to Brussels by train*" comes down to this construction.

$$\lambda w \lambda t\ [[{}^0Do_{wt}\ {}^0John\ {}^0Go] \wedge [{}^0Inst_{wt}\ {}^0Train\ {}^0Go] \wedge$$
$$[{}^0Dir3_{wt}\ {}^0Brussels\ {}^0Go]]$$

It may happen that at another time John will go to Brussels by plane. Then we have

$$\lambda w \lambda t\ [[{}^0Do_{wt}\ {}^0John\ {}^0Go] \wedge$$
$$[{}^0Inst_{wt}\ {}^0Plane\ {}^0Go] \wedge [{}^0Dir3_{wt}\ {}^0Brussels\ {}^0Go]]$$

Wh-questions about John's activity would be, for instance: *What* does John do? *Where* does John go? The content of these questions transforms into constructions like (variables $what \rightarrow \alpha$, $where \rightarrow \iota$)

$$\lambda w \lambda t\ \lambda what\ [{}^0Do_{wt}\ {}^0John\ what]$$
$$\lambda w \lambda t\ \lambda where\ [[{}^0Do_{wt}\ {}^0John\ {}^0Go] \wedge [{}^0Dir3_{wt}\ where\ {}^0Go]]$$

The technique of deducing answers to such Wh-questions has been introduced in Duží, Fait (2020) and (2021). It is an adjusted system of natural deduction with special rules rooted in the rich semantics of natural language and some technical TIL rules stemming from the need to work within a hyperintensional context. Classical natural deduction rules can be applied only to constituents of a construction. For this reason, we need these special

[24] The first proposal of such an analysis of activities with participants has been introduced in Duží (2021). In this paragraph, I introduce a slightly adjusted and corrected analysis. In particular, I do not apply the relation-in-intension *Assign* (an attribute to an activity), as this entity is superfluous and we can obtain a more elegant solution without it.

technical rules.[25] In principle, answers to such Wh-questions are derived by unifying matching terms by means of substituting the values for variables like *what*, *where*, and so like. In our simple example, the answers would be $what = {}^0Go$, $where = {}^0Brussels$.

If agent $b$ has in his ontology the specification of all the possible participants of an activity, and if $b$ obtains an incomplete message where some participants are missing, then $b$ can ask his fellow agents to complete the missing pieces of knowledge. For instance, when receiving the first message about John's going to Brussels by train, the agent can send another query message asking from *where* does John go to Brussels. To this end, we apply the method of analysis of Wh-questions, as introduced above. The content of the query is then this.

$$\lambda w \lambda t\, \lambda d\, [[{}^0Do_{wt}\, {}^0John\, {}^0Go] \wedge [{}^0Inst_{wt}\, {}^0Train\, {}^0Go] \wedge [{}^0Dir1_{wt}\, d\, {}^0Go] \wedge [{}^0Dir3_{wt}\, {}^0Brussels\, {}^0Go]]$$

A possible answer to this Wh-question is the message with this content.

$$\lambda w \lambda t\, [[{}^0Do_{wt}\, {}^0John\, {}^0Go] \wedge [{}^0Inst_{wt}\, {}^0Train\, {}^0Go] \wedge [{}^0Dir1_{wt}\, {}^0Prague\, {}^0Go] \wedge [{}^0Dir3_{wt}\, {}^0Brussels\, {}^0Go]]$$

The answer is obtained by substituting Prague for the variable $d$ using the agents' knowledge base.[26] In case there are two or more actors of the activity, we can apply the relation-in-intension $Do'/(o(o\iota)\alpha)_{\tau\omega}$. For instance, the sentence "*John and Tom go to Brussels by plane on April 1st*" is furnished with this analysis.

$$\lambda w \lambda t\, [[{}^0Do'_{wt}\, \lambda x\, [[x = {}^0John] \vee [x = {}^0Tom]]\, {}^0Go] \wedge [{}^0Inst_{wt}\, {}^0Plane\, {}^0Go] \wedge [{}^0Dir3_{wt}\, {}^0Brussels\, {}^0Go] \wedge [{}^0Time_{wt}\, {}^0April1\, {}^0Go]]$$

The above sentence is underspecified, as it is not clear whether John and Tom are going on their own or together. Yet, the analysis is unambiguous,

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25 See, for instance, Duží,Marie, Jespersen, B. (2015) and Jespersen, B., Duží,Marie (2022), where the rules for existential quantification into hyperintensional contexts have been introduced.

26 For details on deducing answers to Wh-questions by applying the system of natural deduction adjusted to TIL, see Duží, Fait (2021).

as John and Tom are the two actors of *the same activity.* Hence, they are going together. If they went each on their own, it would be two different activities with different actors, even if the other participants were identical.[27]

### *4.2 Agents' activities in past or future*

Another advantage of this approach is this. Since in TIL, we have two modal parameters, time and possible worlds, we can easily specify activities executed in *past* or *future* and model the *dynamic behaviour* and reasoning of agents. If an activity was executed in the past or will be executed in future, the sentence should contain a reference to the *time* when this or that happened or will happen. For instance, the sentence "*John will go to Brussels by plane*" receives this analysis.

$$\lambda w \lambda t\, \exists t'\ [[{}^0Do_{wt'}\ {}^0John\ {}^0Go] \wedge [t' > t] \wedge$$
$$[{}^0Inst_{wt}\ {}^0Plane\ {}^0Go] \wedge [{}^0Dir3_{wt}\ {}^0Brussels\ {}^0Go]]$$

Note that the attributes *Inst* and *Dir3* are extensionalised with respect to time $t$ of evaluation rather than to time $t' > t$, as we assign these attributes now. The situation can change; of course, John can later vote for a car, for instance. In such a case, the sentence is not true.

Anyway, the piece of information conveyed by the sentence seems to be incomplete, as one is tempted to ask, "*When* will John go to Brussels?" It is so because sentences in the past or future should contain a constituent referring to *time* $T \rightarrow (o\tau)$, the time interval when this or that happened or will happen. In such a case, the sentence is associated with a *presupposition* that the current time $t$ is in the proper relation with respect to $T$. Roughly, it means that for sentences in future, $t$ comes before the end of the reference time $T$, while for sentences in past, $t$ comes after $T$; if it is not so, then the proposition denoted by the sentence has a truth-value gap. For instance, the sentence "John will go to Brussels on January 1st, 2023" can be true or false till January 1st, 2023, 24:00. Later, it has no truth value. Involving presupposition is reasonable, of course. Imagine a situation when

[27] I am grateful to the anonymous reviewer for this remark, which lead me to the specification of an activity that is not ambiguous.

(a) asks, "Shall we meet today at 5 p.m.?" using an SMS message, and (b) reads the message later than 5 p.m. Then (b) cannot answer Yes or No. Instead, (b) answers by negating the presupposition, e.g., "Sorry, it is later than 5 p.m. now".

In English, simple past and present perfect are distinct tenses, and we should be able to differentiate them (similarly for simple future and future perfect tenses). While the simple past tense is used for the activities in past that have been finished in past, the present perfect tense is used for past actions that are related to or continue into the present. Detailed analysis of sentences in present perfect tense can be found in Tichý (1980) or Duží et al. (2010, 2.5.2). Briefly, using simple past, the time $t$ of evaluation must be greater than the end of the reference time interval $T$, while for present perfect $t$ must be greater than the beginning of this interval.

Moreover, the sentence can also convey information on the *frequency* of the activity to be executed in the reference time $T$ like 'twice', 'always', 'all the time since', 'for the whole year'. Tichý (1980) introduces a detailed analysis of such sentences in all English tenses. Tichý's analysis is difficult to understand because Tichý applies the *singulariser* function to a singleton typed as containing a truth value in order to make the set fail to deliver a truth value in case the associated presupposition is not satisfied.[28] Tichý's analysis is analogous to what the computer scientist would call an *imperative* rather than a *declarative* analysis. The downside to an imperative analysis is that it may conceal flaws that rear their head only when the analysis is applied to extreme situations. Yet there is an elegant alternative that uses the 'if-then-else' connective proposed by Duží (2010).[29] The author demonstrates here the method of a fine-grained analysis of such sentences equivalent to Tichý's approach but easier to read. In the paper, a general analytic schema for sentences that come associated with a presupposition is presented. To this end, a strict definition of the *If-then-else-fail* function that complies with the compositionality constraint is utilised. In this paper, I am going to apply this solution. Summarising briefly, consider a sentence $S$ with a presupposition $P$. It encodes a meaning procedure, the evaluation of which can be described as follows:

[28] The same method is reproduced in Duží et al. (2010, 2.5.2).

[29] See also Duží (2019b).

> In any $\langle w, t\rangle$-pair of evaluation, *if* $P_{wt}$ is true *then* evaluate $S_{wt}$ to produce a truth value, *else fail* to produce a truth value.

To formulate the schema rigorously, we need to define the *if-then-else-fail* function. First, we define the *if-then-else* function. Here is how. The procedure encoded by "If $P$ ($\rightarrow$ ο) then $C$ ($\rightarrow \alpha$), else $D$ ($\rightarrow \alpha$)" is a two-phase procedure that produces a (strict) function of type ($\alpha$ο$*_n*_n$). Its definition decomposes into two phases.

*First*, select a construction to be executed based on a specific condition $P$. The choice between $C$ and $D$ is specified by this Composition:

$$[{}^0\text{ι}^* \lambda c\ [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]]$$

Types: $P \rightarrow$ ο $v$-constructs the condition of choice between the execution of $C$ or $D$, $C \rightarrow *_n$, $D \rightarrow *_n$, ${}^2C$, ${}^2D \rightarrow \alpha$; $c \rightarrow *_n$; $\text{ι}^*/(*_n\,(\text{ο}*_n))$: the singularizer function that associates a singleton of constructions with the only element of this singleton, and is otherwise (i.e. if the set is empty or many-valued) undefined.

If $P$ $v$-constructs **T** then the variable $c$ $v$-constructs the *construction* $C$, and if $P$ $v$-constructs **F** then the variable $c$ $v$-constructs the *construction* $D$. In either case, the set constructed by

$$\lambda c\ [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]$$

is a singleton and the singularizer $\text{ι}^*$ returns as its value either the construction $C$ or the construction $D$.[30]

*Second*, the selected construction is executed; therefore, Double Execution must be applied:

$$^{\mathbf{2}}[{}^0\text{ι}^* \lambda c\ [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]]$$

30 Note that in this phase $C$ and $D$ are not constituents to be executed; rather they are merely supplied as objects to be selected by the variable $c$. This is to say that in TIL constructions themselves can be objects to be operated on, and without this *hyperintensional* approach we would not be able to define the *strict* function *if-then-else*. For the difference between constructions occurring in the *displayed* and *executed mode*, see, for instance, Duží (2019).

As a special case of the *if-then-else-fail* function, *no* construction $D$ is to be selected whenever $P$ is not satisfied. Thus, the definition of the *if-then-else-fail* function of type $(\alpha o *_n)$ is this:

$$^2[^0\daleth^* \lambda c\ [P \wedge [c = {}^0C]]]$$

Indeed, if $P$ $v$-constructs $\mathbf{F}$, then the class constructed by $\lambda c\ [P \wedge [c = {}^0C]]$ is empty so that the singularizer function does not return as its value any construction. As a result, according to Def. 1, both the composition $[^0\daleth^* \lambda c\ [P \wedge [c = {}^0C]]]$ and its Double Execution are $v$-improper. Applying this definition to the case of an empirical presupposition, we obtain this. Let $P/*_n \rightarrow o_{\tau\omega}$ be a construction of a presupposition of $S/*_n \rightarrow o_{\tau\omega}$. Furthermore, let $c/*_{n+1} \rightarrow *_n$, $^2c \rightarrow o$. Then the type of the *if-then-else-fail* function is $(oo*_n)$ and its definition comes down to this construction:

$$\lambda w \lambda t\ [^0\textit{if-then-else-fail}\ P_{wt}\ {}^0[S_{wt}]] = \lambda w \lambda t\ {}^2[^0\daleth^*\ \lambda c\ [P_{wt} \wedge [c = {}^0[S_{wt}]]]]$$

Instead of the above definition, I use the abbreviated notation to make the *general analytic schema* easier to read:

$$\lambda w \lambda t\ [\textit{if}\ P_{wt}\ \textit{then}\ S_{wt}\ \textit{else fail}].$$

For instance, the truth conditions of the sentence "*John will go to Brussels by plane in 2023*" presuppose that the current time $t$ in which the truth conditions are being evaluated comes before 2023. In other words, the year 2023 comes in future with respect to time $t$. If it is not so, the sentence has *no truth value.* Thus, we have

$$\lambda w \lambda t\ [\textit{If}\ [^0\textit{Future}_t\ {}^0\textit{2023}]\ \textit{then}\ [[\exists t'\ [^0\textit{Do}_{wt'}\ {}^0\textit{John}\ {}^0\textit{Go}] \wedge [^0\textit{2023}\ t']] \wedge$$
$$[^0\textit{Inst}_{wt}\ {}^0\textit{Plain}\ {}^0\textit{Go}] \wedge [^0\textit{Dir3}_{wt}\ {}^0\textit{Brussels}\ {}^0\textit{Go}] \wedge [^0\textit{Time}_{wt}\ {}^0\textit{2023}\ {}^0\textit{Go}]]$$
$$\textit{else fail}]$$

The analysis can also account for the frequency of the activity to be executed in the reference time interval $T$. The general analytic schema for sentences $S$ in future tenses is this.

$$\lambda w \lambda t\ [^0\textit{Future}_t\ [^0\textit{Frequency}_w\ S]\ {}^0\textit{In-Time}] =$$
$$\lambda w \lambda t\ [\textit{If}\ [^0\textit{In-Time} >_\tau t]\ \textit{then}\ [[^0\textit{Frequency}_w\ S]\ {}^0\textit{In-Time}]\ \textit{else fail}].$$

Here $>_\tau$ means that the reference interval *In-Time*/$(o\tau)$ comes after time $t$, *Future* receives the same type as *Past* (which is applied for sentences in past tenses), that is $((o(o(o\tau))(o\tau))\tau)$; *S* is the proposition to be evaluated and *Frequency* is the frequency of time intervals in which the proposition *S* takes the truth-value **T** in world *w.* Hence, the modifier *Frequency* is of type $((o(o\tau))o_{\tau\omega})_\omega$. The schema for sentences in past tenses is similar; it differs only by applying the constituent *Past* instead of *Future.*[31]

If John's activity of going to Brussels by plane in 2023 will be twice a month, by applying the above schema, we obtain this construction.

$$\lambda w\lambda t\ [If\ [^0 2023 >_\tau t]\ then\ [[^0 Twice\text{-}month_w\ \lambda w\lambda t\ [[^0 Do_{wt}\ ^0 John\ ^0 Go] \wedge [^0 Inst_{wt}\ ^0 Plain\ ^0 Go] \wedge [^0 Dir3_{wt}\ ^0 Brussels\ ^0 Go]]]\ ^0 2023]\ else\ fail]$$

Detailed analysis of *Frequency* can also be found in Duží et al. (2010, §2.5.2) or Duží (2010).

### *4.3 Agents' learning new concepts*

As mentioned above, agents can *learn* by experience. They are "born" with a minimal ontology of concepts, which is gradually extended during the agents' life cycle.[32] When agent *a* receives a message from agent *b* containing a concept *C* not contained in *a*'s ontology, *a* does not understand the message. In such a case, agent *a* answers to *b* by sending a query message asking for a *refinement* (i.e. a definition or explication utilising simpler concepts) of the unknown concept *C.* In this way, agents learn new concepts and share their knowledge.[33] To this end, we introduce two 'instructions over concepts', i.e. these relations-in-intension:

[31] A detailed analysis of particular kinds of tenses can be found in (Duží et al. 2010, §2.5.2).

[32] Concept is defined in TIL as a closed construction in its normal form. For details, see Duží et al. (2010, §2.2).

[33] Similar conception has been applied in (Číhalová et al. 2010). In Menšík et al. (2019), the authors introduce the method of refining or explicating atomic concepts by molecular ones using machine learning techniques adjusted to natural language processing. In this way, the agents can learn not only by asking their fellow agents, but also by exploring their environment, in particular by obtaining new pieces of knowledge from the huge amount of text data that are in our disposal.

*Unrecognized*/$(o*_n)_{\tau\omega}$: a property of a concept that an agent does not know it;

*Refine*/$(*_n*_n)_{\tau\omega}$: an empirical function that assigns to a concept $C$ another concept $D$ such that $D$ is a refinement of $C$.

To adduce an example, consider a short communication between agents (a) and (b):

(a) The Incan people used khipu for recording pieces of knowledge.

(b) I do not recognise khipu (I don't know what 'khipu' means, what does 'khipu' stand for.)

(a) Khipu is a recording device fashioned from knotted strings. It had been historically used by a number of cultures in the region of Andean South America, in particular by the Incan people, but also by the ancient Chinese, Tibetans and Japanese.

For the sake of simplicity, I analyse here only the first three sentences and ignore the last one 'It had been historically used by a number of cultures in the region of Andean South America, in particular by Incan people, but also by the ancient Chinese, Tibetans and Japanese.'.

In order to make the content of the agent's (a) first message clear and easier to analyse, let me slightly reformulate the sentence: "There were Incan people who did the activity of recording pieces of knowledge by means of khipu." Here we can use the simple past because the message does not contain any reference time when this activity used to be done, and it definitely stopped being done a long time ago. Thus, we obtain

$$\lambda w \lambda t\ \exists u\ [[u < t] \wedge \exists x\ [[[{}^0 Incan\ {}^0 People]_{wu}\ x] \wedge [{}^0 Do_{wu}\ x\ {}^0 Record]] \wedge [{}^0 Inst_{wt}\ {}^0 Khipu\ {}^0 Record] \wedge [{}^0 Pat_{wt}\ {}^0 Knowledge\ {}^0 Record]]$$

Types. $t,u \rightarrow \tau$; $x \rightarrow \iota$; *Incan*/$((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$: property modifier; *People, Khipu*/$(o\iota)_{\tau\omega}$; *Record(ing)*/$\alpha$: activity; *(pieces of) Knowledge*/$(o*_n)$.

Agent's (b) asking for a definition or refinement of the khipu concept is analysed simply as

$$\lambda w \lambda t\ [{}^0 Unrecognized_{wt}\ {}^{00} Khipu]$$

Gloss. The atomic concept ${}^0Khipu$ has not been recognised; therefore, this very construction must be supplied by another Trivialisation.

Finally, the content of the agent's (a) refining message comes down to this construction.

$$\lambda w\lambda t\ [{}^0Refine_{wt}\ {}^{00}Khipu] = {}^0[\lambda w\lambda t\ \lambda x\ [[[{}^0Recording\ {}^0Device]_{wt}\ x] \wedge \exists y\ [[[{}^0Knotted\ {}^0String]_{wt}\ y] \wedge [{}^0Fashioned\text{-}from_{wt}\ x\ y]]]]$$

Additional types. $x, y \rightarrow \iota$; $Recording, Knotted/((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$: property modifiers; $Device, String/(o\iota)_{\tau\omega}$; $Fashioned\text{-}from/(o\iota\iota)_{\tau\omega}$.

Gloss. Again, a slightly reformulated but equivalent sentence is analysed above, namely this: *Khipu* is a property of individuals $x$ such that $x$ is a recording device and there are individuals $y$ such that they are knotted strings and $x$ is fashioned from $y$. It is harmless here not to seek a strictly literal analysis.

Note that here we again utilise *hyperintensional* features of TIL. The very *concept*, i.e., the *construction* of the respective entity, is asked for refining. An agent who is asking for refinement wants to obtain more detailed *instructions* so that they would understand the message. And this instruction, i.e. procedure, is an object to deal with here rather than the product of the procedure.

## 5. Conclusion

In this paper, we dealt with agents' dynamic activities specified in different tenses. To this end, the linguistic and logical analysis of Wh-questions has been utilised. After a brief introduction to the fundamentals of our background theory of Transparent Intensional Logic (TIL), the logical analysis of Wh-questions and answers in TIL has been illustrated by examples of agents' communication in TIL. Dynamic aspects of agents' reasoning, including messages on participants of activities specified in different tenses and agents' learning by messaging in mutual communication with their fellow agents, have been analysed and demonstrated by examples. The main novelty of the paper is a detailed analysis of agents' activities in present, past and future, specified with reference to the time when the activity happened or will happen to be done together with the frequency of the activity in the reference time.

Further research will concentrate on a still more detailed analysis of messages in different grammatical tenses, presuppositions of such messages, and on dynamic aspects of agents' activities. Here we will also apply the results obtained in the application of Gentzen's natural deduction adjusted for TIL so that these methods can be integrated into one intelligent system.

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